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U.S. Army Coastal Engineering Research Center

DEVELOPMENT OF A METHOD FOR NUMERICAL CALCULATION OF WAVE REFRACTION

TECHNICAL MEMORANDUM NO. 6

DEPARTMENT OF THE ARMY
CORPS OF ENGINEERS

DEVELOPMENT OF A METHOD FOR NUMERICAL CALCULATION OF WAVE REFRACTION

by

W. Harrison

and

W. S. Wilson

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FOREWORD

A knowledge of the energy influx along a shoreline is of considerable importance to an understanding of many beach processes and long-term trends in beach responses. This study represents a step toward the development of a rapid method for routine determinations of wave power along a shoreline, using observed or hindcast deep-water wave characteristics and high-speed computer programs for the calculation of wave refraction. Specifically treated here is a computer program and procedure for the construction of wave rays. An example of the method is presented in which wave rays are brought from deep water into the Atlantic shoreline of the City of Virginia Beach, Virginia. Detailed explanations of the computer programs used, instructions for their operation, and sample inputs and outputs are given in appendices. A series of suggestions is also given for improvement of the method presented.

This report was prepared by Dr. Wyman Harrison, Associate Marine Scientist, Virginia Institute of Marine Science, in pursuance of Contract DA-49-055-CIV-ENG-64-5 with the Coastal Engineering Research Center, and in collaboration with Mr. W.Stanley Wilson, formerly a graduate student at the Institute.

The study was supported by the Coastal Engineering Research Center (formerly the Beach Erosion Board of the Corps of Engineers). Computing Centers at Northwestern University, the College of William and Mary, and NASA (Langley Field, Virginia) extended every cooperation. Lieutenant G. Griswold, Oregon State University, first suggested to the authors the use of numerical methods for the calculation of wave refraction. The computer program used in this report for calculating wave refraction was extensively modified from a program under development in 1963 by Lieutenant G. Griswold and Mr. F. Nagle of the U. S. Navy Weather Research Facility, Norfolk, Virginia, and Mr. E. Mehr of New York University. Mr. J. Gaskin of IBM and LCDR C. Barteau of the U. S. Navy Weather Research Facility aided in adapting the Griswold-Mehr program for use on the 7094 computer, while Mr. J. Curran and Mr. R. Libutti of IBM, aided in attempts to adapt the program to the 1620 computer. Both programs were considerably revised by Wilson prior to their presentation in the appendices of this study.

The authors are indebted to Professor W. C. Krumbein of Northwestern University, Consultant to the Coastal Engineering Research Center, and to Mrs. Betty Benson of the Northwestern University Computing Center, for making available the surface-fitting program that is used in the PRMAT subprogram and SURFCE subroutine of the wave-refraction program. Professor B. R. Cato of the Mathematical Department of the College of William and Mary, provided helpful advice at various stages of the study.

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ABSTRACT

Steps in wave-ray construction are as follows:

1. Select wave periods and approach angles for each series of rays to be constructed.

2. Prepare a grid of depth values for the area of investigation.

3. Use a computer program to obtain a table of water depths and related wave velocities for each wave period selected in (1).

4. Make a grid of wave-velocity values for each wave period selected in (1), for the area of investigation, using a second computer program which takes as input the depth grid of (2) and the appropriate depth-velocity table of (3).

5. Derive, using a third computer program, matrices for use in the linear interpolation scheme of the computer program of (6).

6. Calculate, for each wave period specified in (1), the points along a wave ray using a computer program which takes for input:

   a. The appropriate velocity grid of (4).

   b. The matrices of (5).

   c. The origin points and approach angles of (1) for given wave rays.

[1]Present address. Study completed while at the Virginia Institute of Marine Science.

A linear-interpolation scheme (using the least-squares method) is used in determination of wave velocity at a given point along a ray. Ray curvature is then calculated at this point and an iteration procedure is solved to obtain the position of the next point. The ray terminates at the shore or grid border.

An example of usage of the programs is presented in which 52 rays for waves of 4- and 6-second period, from six different deep-water origin points, are brought into the Atlantic shoreline of the City of Virginia Beach, Virginia.

The procedure outlined is in the developmental stage, and suggestions for improvements are given that should offer a quick, accurate, and objective method of constructing wave rays.

INTRODUCTION

Background of Project

An ultimate understanding of the changes in topography of a given shoreline will be derived from a considerably fuller knowledge of the input and transformation of energy along the strand than is currently available. Because by far the greatest amount of energy expended in the beach-ocean-atmosphere system is associated with breaking waves in the surf zone, it becomes of the greatest importance to evaluate the long-term distribution of wave energy along the shoreline. This energy distribution depends mainly upon the energy of the original deep-water waves, as modified by refraction when the waves move shoreward through shallow water.

It was with these general considerations in mind that the authors embarked upon a study of wave refraction at Virginia Beach, Virginia, where previous studies (cf. U. S. Congress, 1953; Harrison and Wagner, 1964) suggested the desirability of determining wave power distributions. The authors were armed with an "operational" computer program said to be capable of rapid and accurate calculation of wave-ray paths. As in the case of many "operational" computer programs (cf. the cogent comments of Adams, 1964), the authors soon discovered that they were either in difficulty simply trying to make the program operate, or that they were not in agreement with certain of the logical steps involved. It soon became apparent that the entire method needed to be reviewed and revised.

The emphasis here on development of a numerical method for calculation of wave refraction does not mean that the long-range goal of

assessing wave-power distributions along shorelines has been set aside. The authors feel, moreover, that the high-speed method ultimately adopted for calculating wave refraction is of such basic importance to the larger problem that a detailed presentation is warranted at this time.

Earlier Studies of Wave Refraction

The investigations of Krumbein (1944) and Munk and Traylor (1947) were among the first in which steps were taken to assess the link between wave refraction, the energy distribution along the shore, and beach erosion or deposition. These workers constructed wave-refraction diagrams for selected deep-water wave periods by hand-drawn methods, a time-consuming process that at best permits only partial assessment of the effect of a spectrum of waves on shore erosion.

About ten years ago, Pierson, Neumann, and James (1953) considered wave refraction effects in some detail and concluded (1953, p. 186) that to use only significant height and the average "period" of the waves in deep water and to refract the waves with these two numbers will lead to totally unrealistic results. These authors went on to outline a method (1953, p. 197) for forecasting wave heights and characteristics at a point in shallow water near a coast. The method involves construction of a large number of ray diagrams by graphical methods.

Except for the great amount of work involved, the method holds promise for a realistic assessment of energy distributions along a shoreline, assuming the deep-water spectra can be adequately approximated. Although more "practical" methods for determining wave refraction (cf. Silvester, 1963) and for assessing wave energy along coasts (cf. Walsh, Reid, and Bader, 1962) have been proposed, the present authors feel that the adequate treatment of the problem of energy inflow must of necessity include large-scale computations. The treatment of wave refraction, for example, should be as thorough as that outlined by Pierson, Neumann, and James (1952, p. 197), and this requires many man hours. The advent of high-speed computers offers the possibility of significant reductions in the time required for construction of refraction diagrams, wave spectral forecasts, and resultant distributions of energy at the shore. The machine methods given below for constructing refraction diagrams represent a step in the attempt to develop a comprehensive computer program for the routine determination of wave energy along shorelines.

Scope of Present Study

It will be assumed that the deep-water wave spectral periods, heights, and directions are known, and that they may serve as input to a computer program for determining wave refraction as part of an overall method such

as proposed by Pierson, Neumann, and James (1953). Examples of the machine refraction of six separate combinations of wave period and direction off Virginia Beach, Virginia, are given to illustrate the method of refracting wave rays. The assumptions and methodology used in constructing the wave rays are fully treated, and suggestions for improving and amplifying the program are then presented.

METHOD

Previous Work

The presently acceptable method for constructing wave-ray diagrams involves the hand method of graphical construction first proposed by Arthur, Munk, and Isaacs (1952) and later repeated in Pierson, Neumann, and James (1953) and U. S. Army, Corps of Engineers (1961) Technical Report No. 4, of the Beach Erosion Board. This procedure for graphical construction of wave rays involves the use of Snell's Law,

$$\frac{\sin \alpha_1}{\sin \alpha_2} = \frac{C_1}{C_2},$$

where for two successive points on a ray, 1 and 2, C = wave velocity, and α = angle between the wave crest and the bottom contours. In the hand-refracting methods, Snell's Law requires the assumption that between points 1 and 2 there exist straight and parallel contours. The computer program for construction of wave rays (that is to be described later) involves the fitting of interpolative surfaces to points of a wave-velocity grid. This program, although requiring certain assumptions of its own, avoids the assumption of straight and parallel contours between successive points on a ray. With this program an expression for ray curvature is solved quite independently of the use of Snell's Law.

The authors' introduction to the possibilities of computer programs for the numerical construction of wave rays was gained originally through discussions with Lieutenant Gale M. Griswold, then of the U. S. Navy Weather Research Facility, Norfolk, Virginia. Ideas on the subject had appeared in mimeographed reports (Mehr, 1961, 1962a, 1962b; Griswold and Nagle, 1962) and a published paper (Griswold, 1963). A computer program, although not actually operational, was provided by Griswold for development in this study. (This program will be referred to henceforth as the Griswold-Mehr program.)

Construction of Wave Rays

Selection of Input. The first step in wave-ray determination begins with the construction of a grid of depth values for the area of interest.

Geographical limits for a grid of depth values take into account the probable origin points for waves of all deep-water (d/L_o>0.5) approach angles of interest. By convention, the grid origin is located in the southwestern corner of the area to be covered, with the X axis extending eastward along the southern border and the Y axis extending northward along the western border. The grid interval is selected such that, in a given cell, bottom contours can be represented by straight and parallel lines. (This representation will be discussed in detail later.)

Actual or interpolated depths at grid intersections are recorded to the nearest foot, and all real depths are made positive. Contours are then drawn of depth values extending several grid units out from shore. At this point, contours that are symmetrical reflections of nearshore bottom contours are drawn on the land, over a perpendicular distance from the shore of two grid units. "Depths" at the grid points on land are then derived in the region of the symmetry contours on land; these "depth" values are made negative. All other land "depths" (that is, those farther than two grid units from the shore) are made zero. (The procedure of assigning negative values for nearshore "depths" on land is required for the fitting of wave-velocity surfaces.) The depth values so obtained are prepared as input to the DISTV program (Appendix B).

Example of Input. An area of the mid-Atlantic Bight (Figure 1) is chosen to illustrate the method of data input and the computation of wave refraction. A specific wave-ray target area in the region covered by the depth grid is the shoreline of the Borough of Virginia Beach in the City of Virginia Beach, Virginia (Figure 1).

The wave input data used here are not representative of any given wave spectrum observed or hindcast. They approximate the 15 most commonly observed combinations of wave height, period, and direction observed at the Chesapeake Lightship (Figure 1). These combinations were determined and condensed for input to the computer at the request of the Coastal Engineering Research Center. The method used in determination of the combinations is described in Appendix G. The authors have used the six wave period -- direction combinations (Tables 1 and 2), extracted from the 15 height-period-direction combinations of Appendix G, merely to illustrate the method of ray construction.

The depth grid (Figure 1) of the example was chosen so that the origin points of six second waves approaching Virginia Beach from 60° and 90° T would be positioned in water depths of greater than 92 feet (d/L_o>0.5 for T = 6 sec.).

Outlines of the 99 X 81 unit depth grid used in the example are shown on Figure 1; the grid origin is located at 76°1.9'W, 36°39.5'N, and the grid interval is 3,040 feet. U. S. Coast and Geodetic Survey (boat-sheet) charts 5988, 5990, 5991, 5992, 5993, and 6595 were used in obtaining

depth values. Where these charts did not offer coverage, depths were picked off charts 1222 and 1227. Smoothed contours of the depth values at grid points appear on Figure 2.

Deep-water starting points and directions for the 52 wave rays of the example are given in Tables 1 and 2, and are shown plotted on Figure 3. An example of the coding of the wave-ray and wave-velocity input data is given under the heading "INPUT" in Appendix D.

Computer Operations

Programs Used. For each wave period selected for the investigation, a table of depths and their associated wave velocities is prepared. This is accomplished by solving the following equation (U. S. Army, Corps of Engineers, 1942; U. S. Navy, Hydrographic Office, 1944; Mason, 1950) by an iteration process:

$$C = \frac{gT}{2\pi} \tanh \frac{2\pi d}{(TC)}$$

where C = wave velocity, g = acceleration due to gravity, T = wave period, and d = water depth. This equation has been programmed, and a sample depth-velocity table has been prepared for T = 4 seconds. (See COMPV in Appendix A.)

As in other wave refraction studies (Pocinki, 1950; Pierson, 1951; Pierson, Neumann, and James, 1953), it is assumed that wave velocity is a function only of water depth and wave period, as expressed by the above equation. Various factors such as bottom friction, percolation, reflection, currents, and winds are considered as having no effect on the refracting waves.

Given the absolute value of a water depth, it is possible to check the appropriate table, which has been previously prepared, for the associated wave velocity. Preparation of an entire velocity grid for each wave period to be studied is then carried out. This procedure has also been programmed, and a portion (from X = 0 to X = 19, and from Y = 0 to Y = 2) of the depth grid described above has been derived for T = 4 seconds. (See DISTV in Appendix B.)

The procedure for determining the wave-velocity value at an arbitrary point in a grid cell involves fitting a plane to the velocity values at the four grid intersections that bound the grid cell in question. This linear surface is fit by the least-squares method, using an equation of the form:

$$C = E_1 + E_2X + E_3Y$$

where C = wave velocity, E's = coefficients, and X,Y are the grid coordinates for the arbitrary point.

With the least-squares method of surface fitting, it is possible to obtain certain matrices which are used each time the equation of a plane is derived for a grid cell. These matrices have been derived in PRMAT. (See Appendix C.) That portion of the surface-fitting procedure which must be carried out each time a linear equation is to be derived is given in SURFCE subroutine of MAIN 1620 (Appendix D) and MAIN 7094 (Appendix E).

Determination of each desired ray for a given period is accomplished by first specifying origin coordinates and an angle of approach. The actual ray is constructed by plotting and connecting the series of successive calculation points (computed by MAIN 1620, Appendix D, or MAIN 7094, Appendix E) which range across the velocity grid until striking the beach or grid margin. (As discussed by Griswold (1963, p. 1722) the rays may be run from the shore seaward, if the construction of a refraction diagram at a point is desired.) Velocity at each point is calculated as mentioned above; ray curvature (FK) is calculated by using the following expression (Munk and Arthur, 1952):

$$FK = \frac{1}{C}\left[\sin A\left(\frac{\partial C}{\partial X}\right) - \cos A\left(\frac{\partial C}{\partial Y}\right)\right]$$

where A = approach angle, as defined in Appendix D.

In order to determine X_{n+1}, Y_{n+1}, A_{n+1}, and FK_{n+1} for calculation point n+1 (with those similar values known for point n), the following equations (Griswold and Nagle, 1962; Griswold, 1963) are solved by an iteration procedure:

$$\Delta A = (FK_n + FK_{n+1})D/2$$

$$A_{n+1} = A_n + \Delta A$$

$$\bar{A} = (A_n + A_{n+1})/2$$

$$X_{n+1} = X_n + D\cos\bar{A}$$

$$Y_{n+1} = Y_n + D\sin\bar{A}$$

where D equals the incremented distance between points n and n+1. (See MOVE subroutine of MAIN 1620, Appendix D, and of MAIN 7094, Appendix E.)

The computers to be used for the programs cited in the appendices are the IBM 1620 (with floating-point hardware and 60K memory) and the IBM 7094. FORTRAN II is the language used for the 1620 programs; FORTRAN IV is used for the 7094 programs. The computer and language for a given program are specified on the first comment card of each program print-out.

The number of digits carried by a computer during internal computations for floating-point numbers (designated by f) and the number of digits carried for fixed-point numbers (designated by k) vary with the different programs. For the 1620 programs, f is given in Columns 2 and 3 and k is given in Columns 4 and 5 of the first card of each program print-out. For the 7094 program, f = 8 and k = 5.

Example of Sample Output. Sample input and output listings are given in Appendix D for three wave rays. These three rays, numbered 1, 2, and 3 in the OUTPUT listings, correspond to rays numbered 33, 32, and 31, respectively, of Table 1 and Figures 2 and 6.

Representation of Output

Possible Methods. Output from the computer programs could be presented in a variety of ways, depending upon the requirements of the investigator. Precision work on caustics, for example, might entail skillful plotting of the values at MAX points with precision drafting techniques. If a rapid analysis of ray paths from deep water to the shore is all that is required, however, the investigator might consider an X-Y plotter attachment for rapid printing of the computer output. For many shore engineering studies, only the final few hundred yards of the wave rays will be of practical value, while for studies of energy or wave-power distribution along a fixed segment of the shore, it is possible that only the terminal points of the rays at some specified distance from shore would be of value for presentation.

Example. The results of the computer refraction of the 52 wave rays of Figure 2 are presented in Figures 3-8, which show only the last portions of the wave rays relatively close to shore. Successive calculation points were plotted on a grid and then connected by a smooth curve. The rays themselves vary from the almost unmodified ones for 4-second waves that approach the beach relatively head on (Figure 5), to the 6-second ones that actually cross (Figure 8) within the limits of the figure.

Attention is drawn to the terminal points of the wave rays; some of these points (such as those of rays Nos. 14, 37, and 52 as shown in Figures 4, 7, and 8, respectively) are closer to the shoreline than others. These variations are due to the fact that a ray terminates because either the next calculation point is in an area of zero or negative velocity, or curvature approximations are not converging. Thus, with a constant incremented distance (D), all rays do not reach a similar distance from the shoreline. Ray No. 24 (Figure 5), on the other hand, makes a pronounced

curve near its terminus that appears out of context when compared to the adjacent wave rays. This is due to a combination of (1) a poor "fit" by the linear-interpolation surface at this point, and (2) poor grid control (i.e., poor representation of depth values recorded on the initial depth grid). Suggestions for elimination of these factors are discussed in the section "Grid Considerations."

PROGRAM DEVELOPMENT

The computer programs presented in the appendices have by no means been refined to the fullest extent. At the moment the following factors may be considered of most importance to their continued development: (1) improvement of the surface-fitting scheme for wave velocity interpolation, (2) improvement of the method of ray-curvature approximation, (3) addition of a provision for changes in grid scale and incremented distance as a ray moves shoreward, (4) refinement of ray-plotting techniques, and (5) testing of the revised program at a suitable place in nature.

Interpolation Surfaces

"Forced-cubic" Interpolation Surface. First mentioned by way of review is the surface-fitting scheme for interpolation of wave velocity used in the Griswold-Mehr program. Their scheme involves fitting a cubic surface that (1) passes exactly through the velocity values at the grid intersections of the given grid cell, and (2) is the cubic surface of best fit (by the least-squares method) to the velocity values at the eight grid intersections closest to and surrounding the four grid intersections of the given cell. This cubic surface is called a "forced-cubic" surface because it is "forced" to pass through the four innermost velocity values. Because it permits the taking of first and second derivatives for use in wave intensity calculations, as explained by Griswold (1963, p. 1720), it was the first surface-fitting program used in this study.

An example of the results of its use appears in Figure 9C. It is obvious from the figure that this method of interpolation is invalid. The forced feature of the surface creates undesired ridges and/or troughs in the velocity surface. Thus, depending upon the location of a given calculation point in a grid cell, the ray can be deflected erratically from a "normal" path. Results such as those exemplified in Figure 9C necessitated a search for an interpolation surface that could adequately portray the general trend of wave velocity change in a given cell.

Quadratic-interpolation Surface. The Griswold-Mehr program was altered by insertion of a quadratic surface of best fit (by the least-squares method) subroutine for preparation of the interpolation surface. In order to derive a quadratic surface, at least six data points are necessary. It was decided to use the velocity values at the closest 12 grid intersections. The results (Figure 9B), however, did not yield a

satisfactory interpolation surface; this was evident when calculation points for a ray had moved within two grid units of the shore. This is because the negative "land" velocity values, used in derivation of the surface, made the general tilt of the surface steeper than indicated by the velocity values at the central four grid intersections. This is shown in Figure 9B where the rays bend, after moving within two grid units of the shore, more than do the rays produced by the linear-interpolation program (Figure 9A) or the rays produced by the graphical-construction method (Figure 9D).

Linear-interpolation Surface. The linear-interpolation programs (MAIN 1620, Appendix D; and MAIN 7094, Appendix E) were developed in order to remedy the excessive tilt created by the quadratic-interpolation program. The advantage offered by the linear-interpolation program is that only the velocity values at the central four grid intersections are needed in order to derive the surface. The assumption, presented under the heading "Selection of Input," stating that the grid interval be selected such that bottom contours in any grid cell be represented by straight and parallel lines, is not actually valid. Although bottom topography may be represented by a plane (i.e., it changes linearly) in a given cell, the associated velocity values may be changing exponentially (i.e., velocity is an exponential function of depth) in the same cell. However, for purposes of this program, it is assumed that the velocity values in a given grid cell can be adequately represented by a plane.

The use of the variable PCTDIF in MAIN 1620 (card number RAYN 19, Appendix D) and MAIN 7094 (card number RAYN 20, Appendix E) serves to give an indication of the relative "fit" of a surface to a given set of velocity values. The output values for PCTDIF (see OUTPUT, Appendix D) give the percent difference between only one of the four velocity values at the nearest four grid intersections and its related value on the plane fit to the same four values. PCTDIF by no means represents the degree of "fit" of the surface to the four values because these percent differences may vary for each of the four values to which a plane is fit. This is especially true in cells where there are great differences between the four velocity values (such as near the shore). PCTDIF does, however, give an estimate of the relative error encountered in the interpolation in a given grid cell.

Just as the quadratic-interpolation program yielded an excessive tilt when the given grid cell in use fell within two grid units of the shore, the linear-interpolation program yields an excessive tilt when the given cell falls within one grid unit of the shore. Therefore, it is expected that PCTDIF will be large in value when the grid cell being interpolated is located near the shore. In view of this fact, the rays run with the linear surface-fitting program should be considered as rough approximations only, when closer than one grid unit to the shore.

Interpolation Surface Versus Graphical Method of Ray Construction. Consideration of the above-mentioned surface-fitting programs led to the choice of the linear surface of best fit as the most suitable one for use in construction of wave rays. In general, it was found that rays run with the linear surface of best fit compared most favorably with rays constructed by graphical methods, and this is the case in the plotting example (Figure 9, A and D). An advantage of the least-squares linear-interpolation method over the graphical method lies in the fact that ray curvature can be computed at a number of points between contours. Aside from the absolute validity of the two methods of wave-ray construction, the computer method is estimated to be many times faster than the hand method when only the terminal points are desired of a large number of rays. Where only a few rays are desired, the hand method is clearly the most rapid and economical. Because the practice of refracting only a few rays for a few wave periods yields totally inadequate information upon which to assess wave heights and energies at the shore, it seems clear that the real considerations are not so much the man-hours involved in depth-grid and program-deck preparation for the computer as they are in the necessity for the more realistic results that can be afforded by the computer construction of wave rays for entire wave spectra.

Future Interpolation Schemes. A more valid interpolation scheme would be obtained if a grid of depth values were input to MAIN 1620 or MAIN 7094 instead of a grid of velocity values. The assumption, that the grid interval be selected such that depth contours in any grid cell be represented by straight and parallel contours, would then be perfectly valid. In this case, a depth value would be obtained from the interpolation surface at a given calculation point. Then, using the procedure used by COMPV (Appendix A), the depth value could be converted into a wave-velocity value. It is noted that, in order to obtain curvature (FK) at a calculation point, $\frac{\partial C}{\partial x}$ and $\frac{\partial C}{\partial y}$ are needed. If a linear-interpolation program were used with an input depth grid, expressions would be available for $\frac{\partial d}{\partial x}$ and $\frac{\partial d}{\partial y}$. The following relationship (derived in Appendix F) could then be used to obtain $\frac{\partial C}{\partial x}$ and $\frac{\partial C}{\partial y}$:

$$\frac{\partial C}{\partial x} = \frac{\partial d}{\partial x} \cdot Z \ , \ \frac{\partial C}{\partial y} = \frac{\partial d}{\partial y} \cdot Z$$

where

$$Z = \frac{1}{k'} \left[\frac{1}{\frac{Ck''}{1+k''C} + \frac{Ck''}{1-k''C} + \ln(1+k''C) - \ln(1-k''C)} \right]$$

In this expression $k' = T/4\pi$ and $k'' = 2\pi/gT$. If a quadratic surface were to be used for interpolation similar relations could be derived to relate $\frac{\partial^2 C}{\partial x^2}$ and $\frac{\partial^2 C}{\partial y^2}$ with $\frac{\partial^2 d}{\partial x^2}$ and $\frac{\partial^2 d}{\partial y^2}$, respectively.

A reason for using a quadratic (or a higher-order polynomial) surface is that the second partial derivatives can be obtained. This fact was used in the Griswold-Mehr program to obtain values for Beta (Munk and Arthur, 1952), the coefficient of refraction, and H/H_0. (As mentioned, however, their interpolation scheme rendered their results invalid.) It is apparent that certain problems with the quadratic (or higher-order polynomial) interpolation schemes must be resolved before such sophisticated parameters as Beta or H/H_0 can be estimated. It should be possible to derive a quadratic surface by heavily weighting the velocity values at the central four grid intersections while still using the surrounding eight velocity values. This procedure may produce a good interpolation surface; if so, it would be quite worth while to calculate estimates of the above parameters.

Ray Curvature Approximation

An entire grid of velocity values is not a smooth and continuous surface when observed as a set of planes in which each grid cell is represented by a specific plane of best fit to its four bounding velocity values. With this in mind, it is not surprising that all curvature approximations (see MOVE subroutine of MAIN 1620, Appendix D; or MOVE subroutine of MAIN 7094, Appendix E) fail to converge to a single value when determining a new point along a ray path. This is especially apparent when adjacent planes present a large discontinuity in wave-velocity values at their common edges. This fact is the reason for the variable MIT included in MOVE subroutine. In case the curvature approximations oscillate among three or more values after 20 iterations, the ray is terminated, as no valid curvature approximation can be made. Although infrequent, sometimes (not shown in OUTPUT, Appendix D) the curvature approximations oscillate between two values. In this case, the message "CURVATURE APPROXIMATED" is included with the output in order that the operator note that the curvature used for the given point was an average of two values. The Griswold-Mehr program did not have such a check on the curvature approximations; the average of the last two approximations after 10 iterations was used as the new curvature value, if the values did not converge. This is another reason for the erratic ray behavior near the shore in Figure 9C.

Grid Considerations

For reasons outlined in a previous section ("Future Interpolation Schemes") assume that a grid of depth values will be input and used for derivation of the interpolation surfaces. It will be necessary in the future program to transfer a ray to a larger scale grid when approaching the shore in order to allow better grid control (i.e., better representation of depth values) in an area where the velocity values are rapidly changing. There is, however, a limit to the maximal scale of a grid as beach and nearshore topography are constantly changing, especially during each storm. Thus the selection of grid interval is a question to be pursued in additional studies.

It will also be necessary to vary the distance (D) incremented between successive calculation points along a ray in order that the ray may approach as close to the shoreline as possible. As an example, the value of D could be assigned such that D = 0.5 when $d/L_o > 0.5$ and $D = d/L_o$ when $d/L_o < 0.5$.

Wave Ray Representation

At present, output must be plotted by hand; and, if there are a large number of rays to be constructed, this can be a tedious and time-consuming process. The use of an X-Y plotter, if adaptable to changing grid scales, would be an ideal scheme for rapid plotting of wave rays. Another scheme might involve the hand or machine plotting of the (numbered) terminal points of the wave rays, along the shore or grid margin.

Testing of Ray Constructions

The absolute validity of the ray constructions (Figures 3-8) is impossible to evaluate without a test in nature. It would be desirable to test the ray constructions in an area where significant refraction of relatively constant-period wave trains occur and where the bathymetry is well known. The use of aerial photography would be an invaluable aid in conducting such a test.

It is possible to test ray constructions by comparison with analytic solutions for wave rays passing over algebraically-described surfaces (Pocinki, 1950). Griswold (1953), in testing the Griswold-Mehr program, used both a straight uniformly-sloping beach and a parabolic bay and found little variation between computed and analytic rays. However, such a theoretical test is not sufficient reason for acceptance of a computer program utilizing an interpolation scheme. In these theoretical tests, an algebraically-described surface of velocity values is input to the computer. The close agreement then noted between computed and analytic rays is due to the fact that the given interpolation scheme can easily and accurately represent the velocity surface when interpolating within any grid cell. In order to adequately test such a computer program, an irregular surface of velocity values must be provided. This necessity supports the need for a test in nature, as mentioned in the previous paragraph.

CONCLUSION

The procedure described in this report, when fully improved using the accompanying suggestions, will constitute a rapid and accurate method of wave ray construction. At the present stage of development, however, the procedure must be accepted with reservation.

REFERENCES

Adams, B. B., 1964, Program pool debated: Geotimes, v. 9, p.4.

Arthur, R. S.; Munk, W. H., and Isaacs, J. D., 1952, The direct construction of wave rays: Trans. Amer. Geophys. Union, v. 33, no. 6.

Griswold, G. M., and Nagle, F. W., 1962, Wave refraction by numerical methods: Mimeo. Rept., U. S. Navy Weather Research Facility, 19 p.

Griswold, G. M., 1963, Numerical calculation of wave refraction: Jour. Geophys. Res., v. 68, p. 1715-1723.

Harrison, W., and Wagner, K., 1964, Beach changes at Virginia Beach, Virginia: U. S. Army, Corps of Eng., Coastal Eng. Res. Center Tech. Memo. (in press).

Johnson, J. W.; O'Brein, M. P.; and Isaacs, J. D., 1948, Graphical construction of wave refraction diagrams: U. S. Navy Hydro. Off. Pub. No. 605, 45 p.

Kaplan, Wilfred, 1952, Advanced calculus: Addison-Wesley Pub. Co., Reading, Mass., 679 p.

Mason, M. A., 1950, The transformation of waves in shallow water: in Proc. First Conf. Coastal Engr., Counc. Wave Res., Berkeley, Calif., p. 22-32.

Mehr, E., 1961, Surf refraction: Prog. Rept. No. 1 to U. S. Navy Weather Res. Fac., Norfolk (Contract No. N189-188-54411A, mimeo. rept.), 4 p.

__________, 1962a, Surf refraction: Prog. Rept. No. 2 (see citation immediately above), 11 p.

__________, 1962b, Surf refractions; computer refraction program: Final Rept. of N. Y. Univ. Statistical Lab. (Coll. Eng., Res. Div.) to U. S. Navy Weather Res. Fac., Norfolk, on Contract No. N189-188-5411A, 21 p.

Munk, W. H., and Traylor, M. A., 1947, Refraction of ocean waves: a process linking underwater topography to beach erosion: Jour. Geology, v. 55, p. 1-26.

Munk, W. H., and Arthur, R. S., 1952, Wave intensity along a refracted ray: in Gravity waves: U. S. Dept. Comm., Nat. Bur. Standards Circ. 521, p. 95-108.

Pierson, W. J., Jr., 1951, The interpretation of crossed orthogonals in wave refraction phenomena: U. S. Army, Corps of Engineers, Beach Erosion Board, Tech. Memo. 21, 83 p.

Pierson, W. J., Jr.; Neumann, G.; and James, R. W., 1953, Observering and forecasting ocean waves: U. S. Navy Hydro. Off. Pub. No. 603, 284 p.

Pocinki, L. S., 1950, The application of conformal transformations to ocean wave refraction problems: Trans. Amer. Geophys. Union, v. 31, p. 856-866.

Silvester, R., 1963, Design waves for littoral models: Jour. Waterways and Harbor Div., Amer. Soc. Civ. Eng., v. 89, WW3, p. 37-47.

U. S. Army, Corps of Engineers, 1942, A summary of the theory of oscillatory waves: Beach Erosion Board Tech. Rept. No. 2, 43 p.

__________, 1961, Shore protection, planning and design: Beach Erosion Board Tech. Rept. No. 4, 241 p.

U. S. Congress, 1953, Virginia Beach, Va., beach erosion control study: 83rd Congress, 1st Session, House Doc. 186, 45 p.

U. S. Navy, Hydrographic Office, 1944, Breakers and surf, principles in forecasting: Pub. No. 234, 54 p.

Walsh, D. E.; Reid, R. O.; Bader, R. G., 1962, Wave refraction and wave energy on Cayo Arenas, Campeche Bank: Texas A. and M, Dept. of Oceanography Ref. 62-6T, 62 p.

Wilson, W. S., 1964, Improving a procedure for numerical construction of wave rays: Masters Thesis, School of Marine Science, College of William and Mary, Williamsburg, Virginia.

TABLE 1.-Computer input specifications for waves of 4-sec period

Ray no.	Ray origin angle for computer (A)	Grid origin (X)	Grid origin (Y)	Direction from which ray travels (°T)
1	240.0	37.50	76.50	30
2	240.0	38.38	76.00	30
3	240.0	39.25	75.50	30
4	240.0	40.12	75.00	30
5	240.0	41.00	74.50	30
6	240.0	41.87	74.00	30
7	240.0	42.74	73.50	30
8	210.0	26.92	39.23	60
9	210.0	27.43	38.38	60
10	210.0	27.94	37.52	60
11	210.0	28.45	36.66	60
12	210.0	28.96	35.80	60
13	210.0	29.47	34.94	60
14	210.0	29.98	34.08	60
15	210.0	30.49	33.22	60
16	210.0	31.00	32.36	60
17	210.0	31.50	31.50	60
18	180.0	21.50	26.50	90
19	180.0	21.50	25.50	90
20	180.0	21.50	24.50	90
21	180.0	21.50	23.50	90
22	180.0	21.50	22.50	90
23	180.0	21.50	21.50	90
24	180.0	21.50	20.50	90
25	180.0	21.50	19.50	90
26	180.0	21.50	18.50	90
27	180.0	21.50	17.50	90
28	120.0	18.81	04.96	150
29	120.0	17.95	04.47	150
30	120.0	17.08	03.97	150
31	120.0	16.22	03.49	150
32	120.0	15.36	02.99	150
33	120.0	14.50	02.50	150

Table 2.-Computer input specifications for waves of 6-sec period.

Ray no.	Ray origin angle for computer (A)	Grid origin (X)	Grid origin (Y)	Direction from which ray travels (°T)
34	210.0	85.50	73.50	60
35	210.0	86.01	72.64	60
36	210.0	86.52	71.78	60
37	210.0	87.03	70.92	60
38	210.0	87.54	70.06	60
39	210.0	88.05	69.20	60
40	210.0	88.56	68.34	60
41	210.0	89.07	67.48	60
42	210.0	89.58	66.62	60
43	180.0	91.50	26.50	90
44	180.0	91.50	25.50	90
45	180.0	91.50	24.50	90
46	180.0	91.50	23.50	90
47	180.0	91.50	22.50	90
48	180.0	91.50	21.50	90
49	180.0	91.50	20.50	90
50	180.0	91.50	19.50	90
51	180.0	91.50	18.50	90
52	180.0	91.50	17.50	90

FIGURES

Text

Appendix G

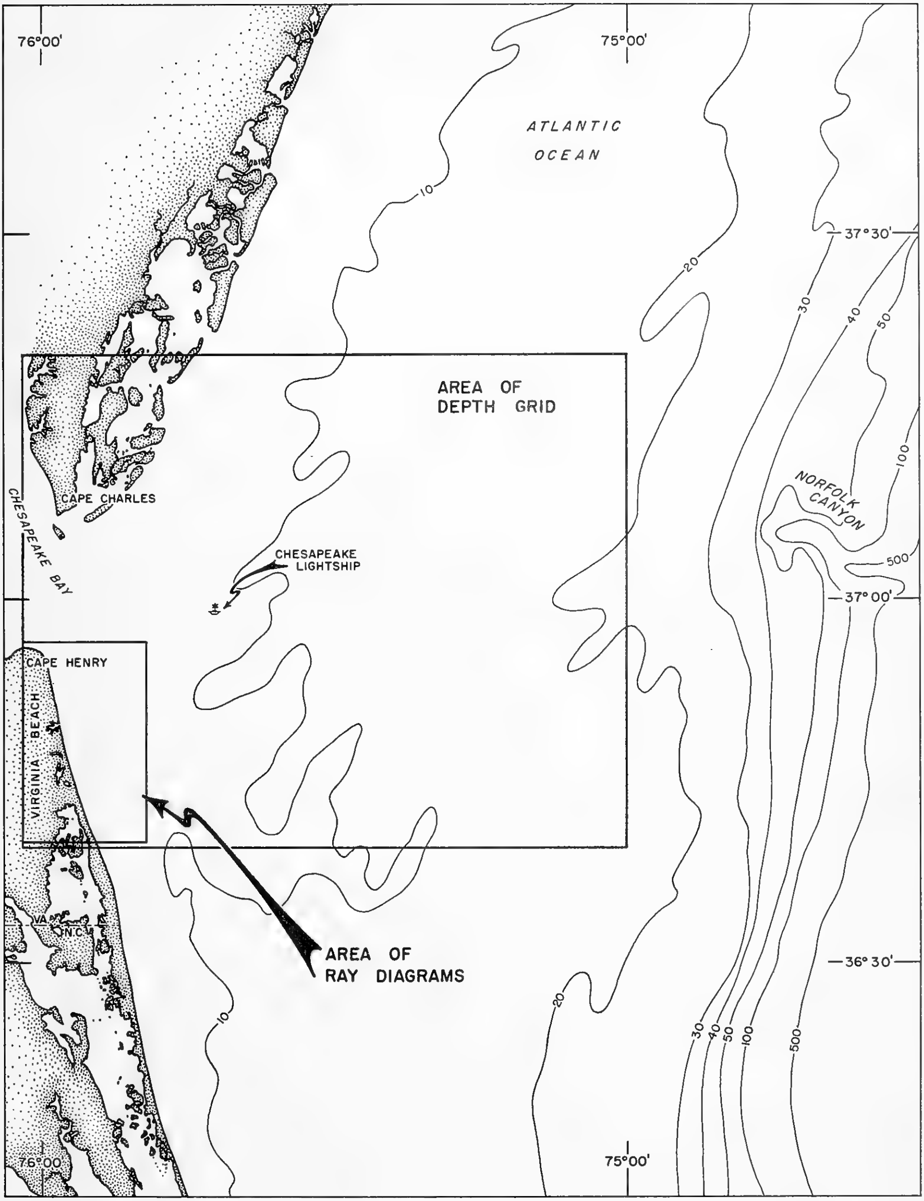
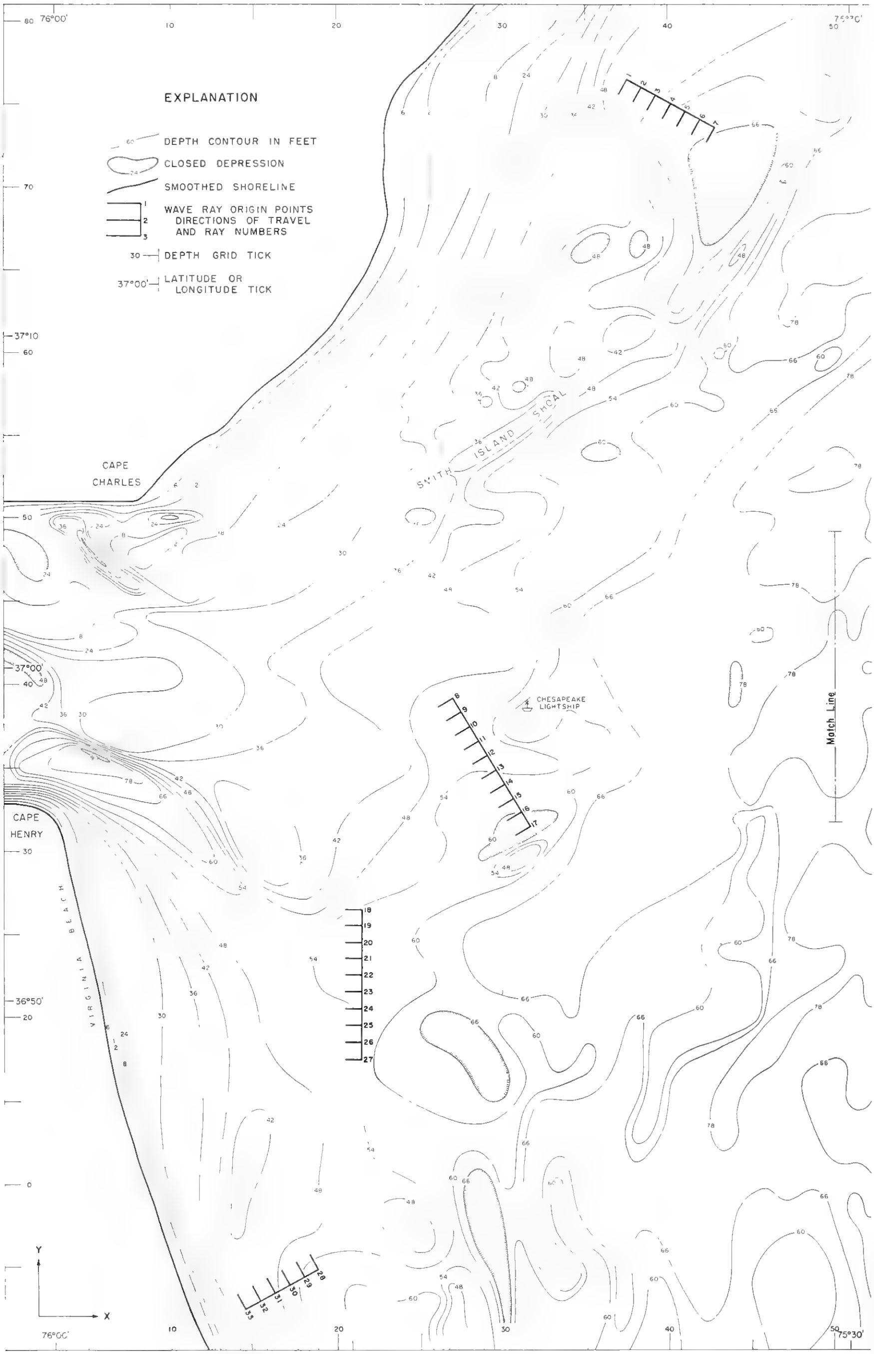


FIGURE I. GENERALIZED DEPTH CONTOURS IN FATHOMS


EXPLANATION
DEPTH CONTOUR IN FEET
CLOSED DEPRESSION
SMOOTHED SHORELINE
WAVE RAY ORIGIN POINTS DIRECTIONS OF TRAVEL AND RAY NUMBERS
DEPTH GRID TICK
LATITUDE OR LONGITUDE TICK
CAPE CHARLES
CAPE HENRY
SMITH ISLAND SHOAL
CHESAPEAKE LIGHTSHIP
VIRGINIA BEACH
Match Line
76°00'
75°30'
37°10
37°00'
36°50'

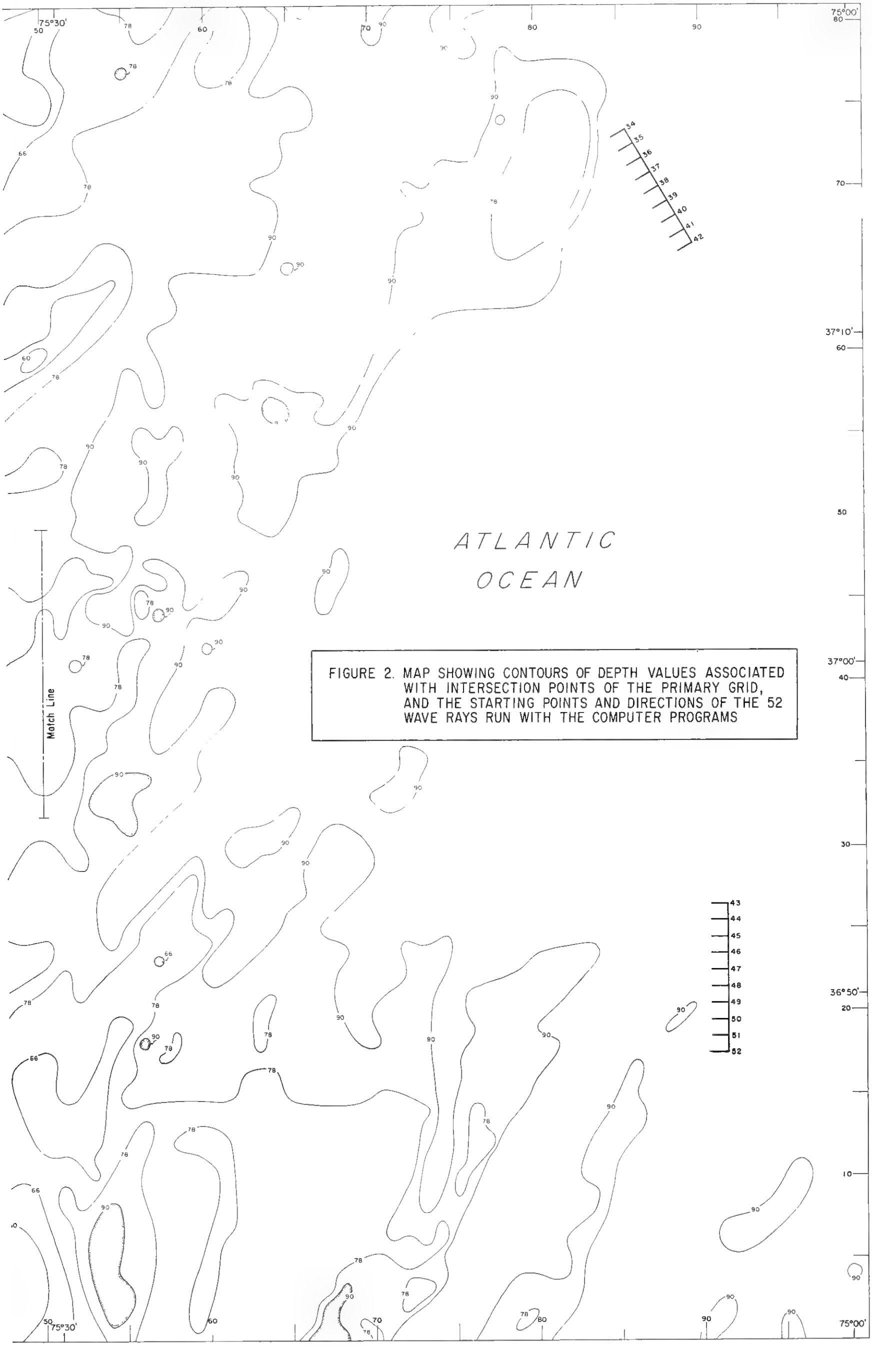


FIGURE 2. MAP SHOWING CONTOURS OF DEPTH VALUES ASSOCIATED WITH INTERSECTION POINTS OF THE PRIMARY GRID, AND THE STARTING POINTS AND DIRECTIONS OF THE 52 WAVE RAYS RUN WITH THE COMPUTER PROGRAMS

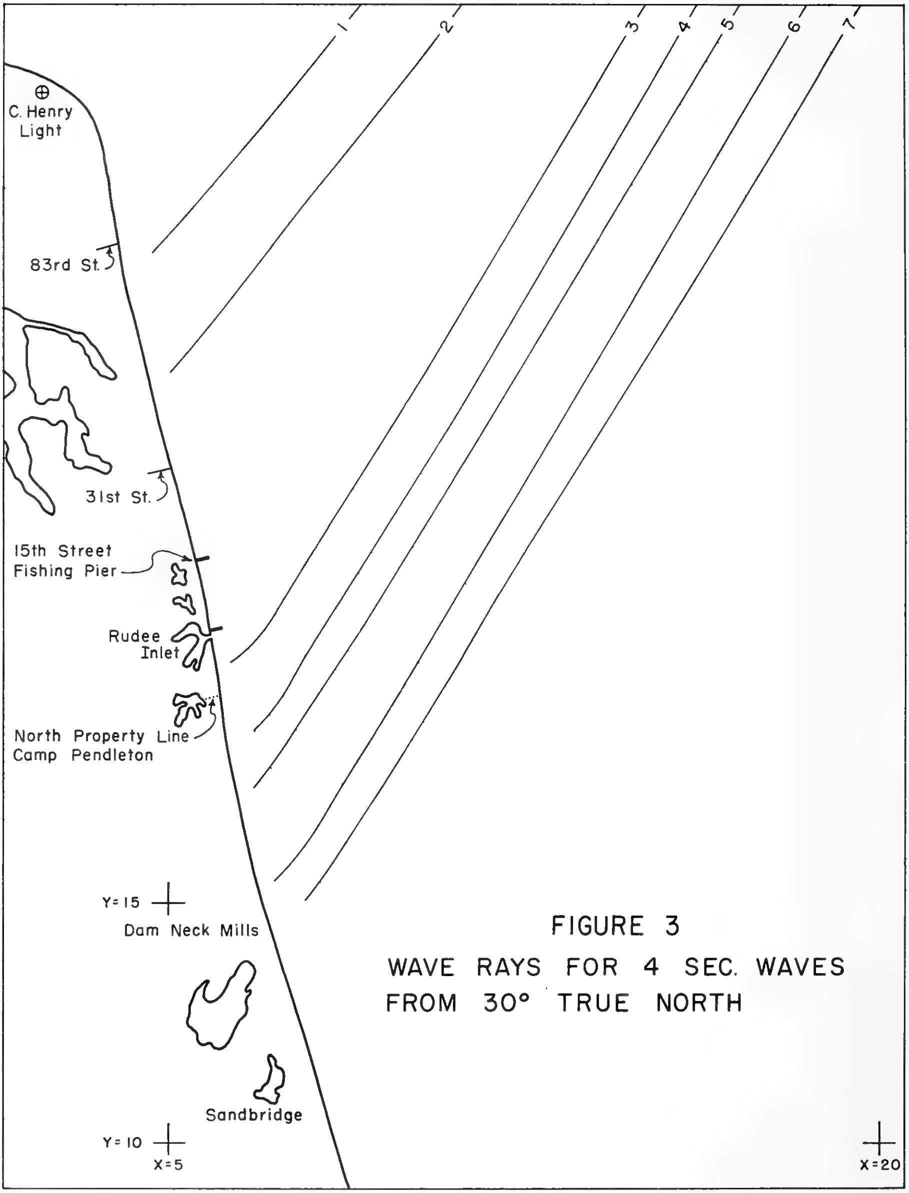


FIGURE 3
WAVE RAYS FOR 4 SEC. WAVES
FROM 30° TRUE NORTH

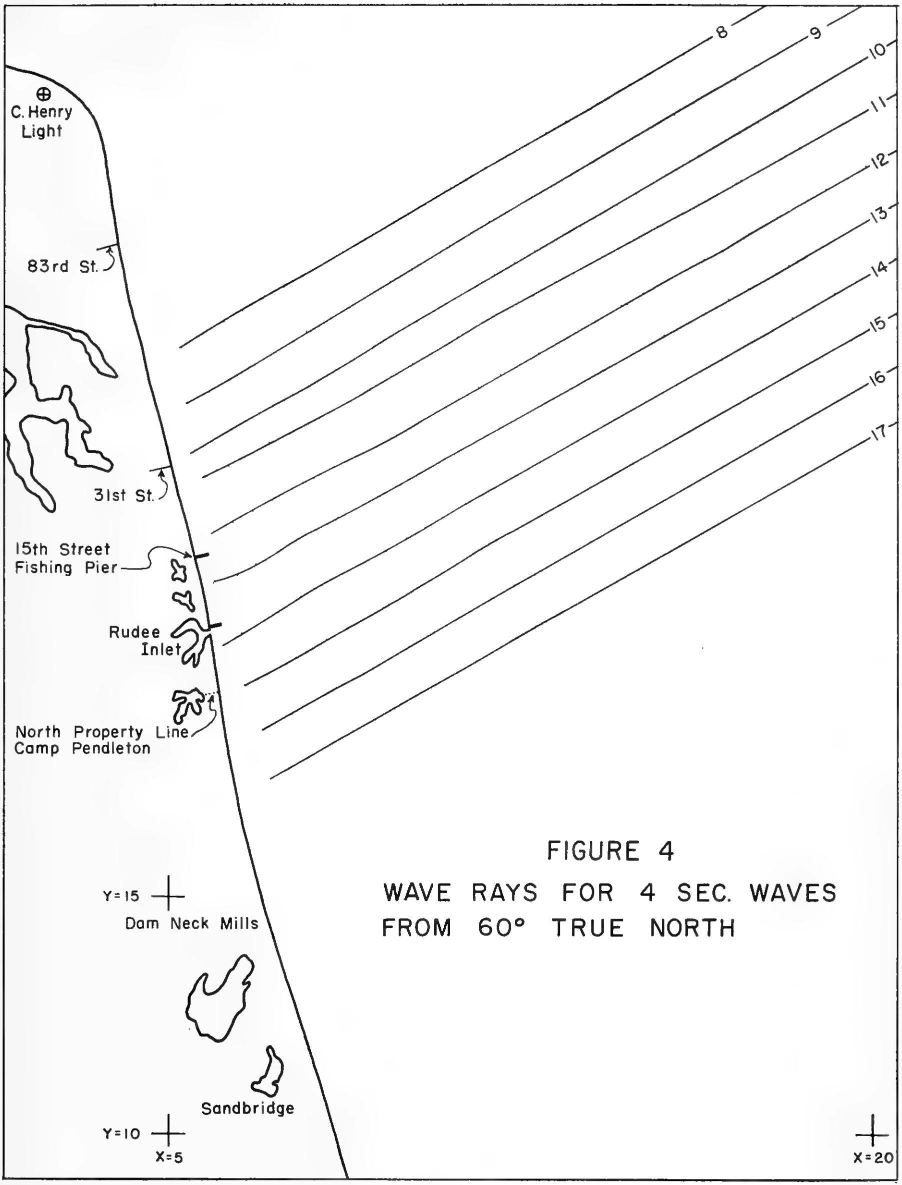


FIGURE 4

WAVE RAYS FOR 4 SEC. WAVES FROM 60° TRUE NORTH

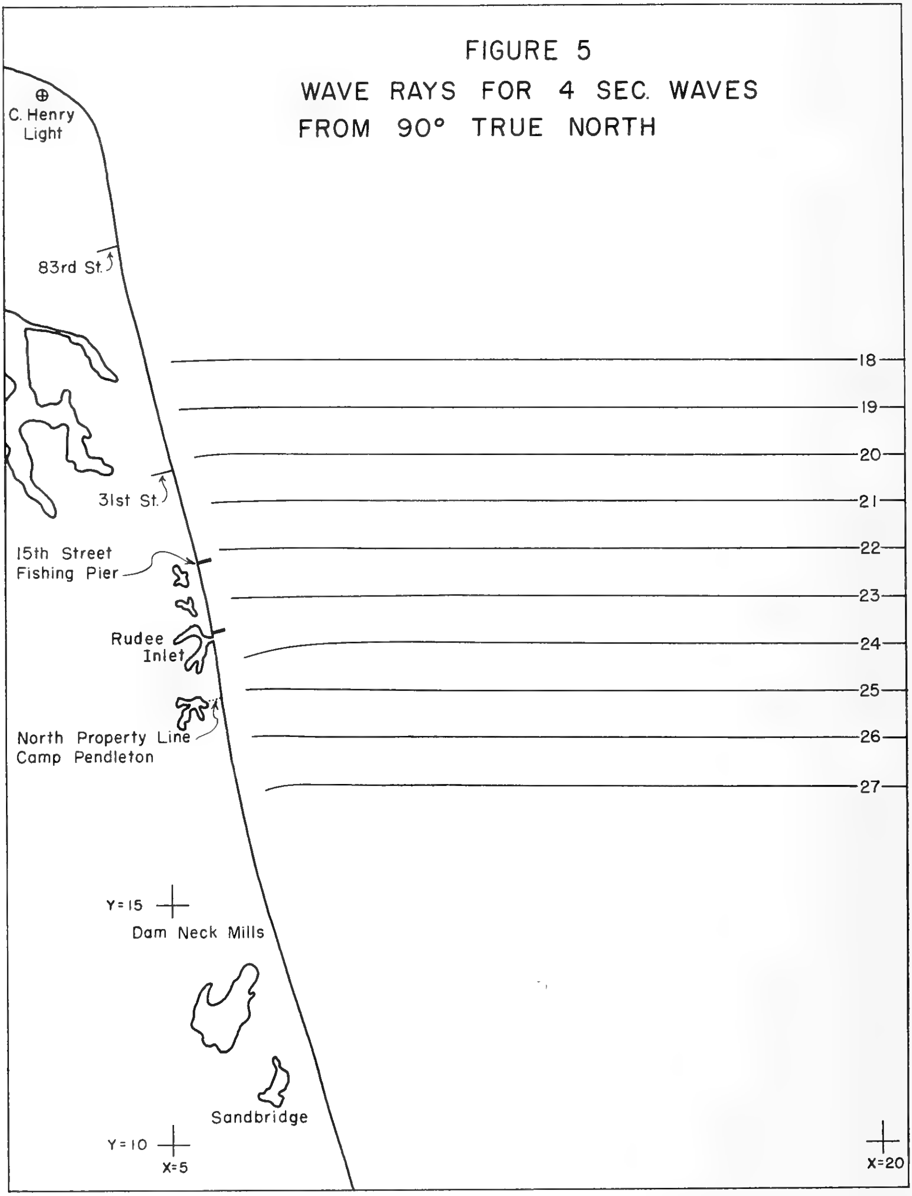


FIGURE 5
WAVE RAYS FOR 4 SEC. WAVES
FROM 90° TRUE NORTH
C. Henry
Light
83rd St.
31st St.
15th Street
Fishing Pier
Rudee
Inlet
North Property Line
Camp Pendleton
Y=15
Dam Neck Mills
Sandbridge
Y=10
X=5
X=20
18
19
20
21
22
23
24
25
26
27

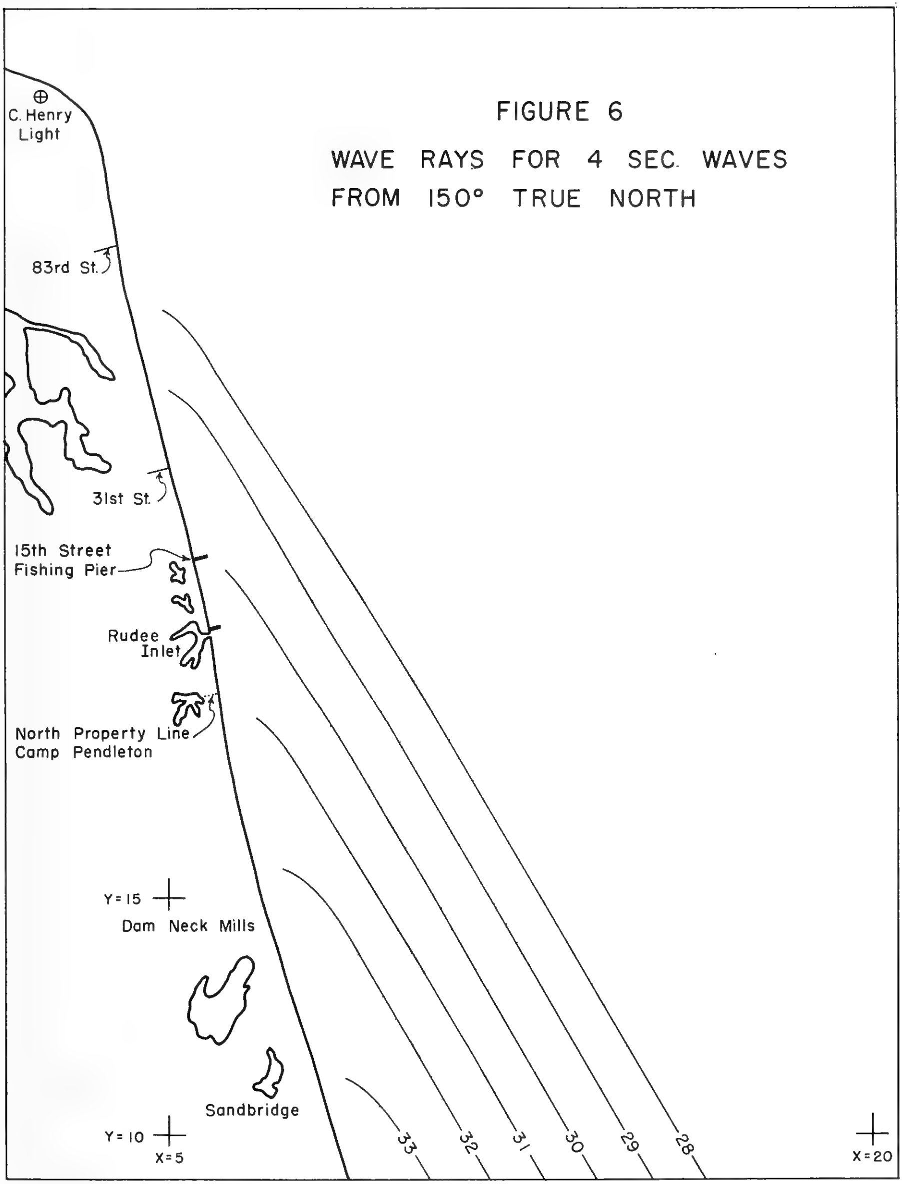


FIGURE 6
WAVE RAYS FOR 4 SEC. WAVES
FROM 150° TRUE NORTH
C. Henry
Light
83rd St.
31st St.
15th Street
Fishing Pier
Rudee
Inlet
North Property Line
Camp Pendleton
Y=15
Dam Neck Mills
Sandbridge
Y=10
X=5
33
32
31
30
29
28
X=20

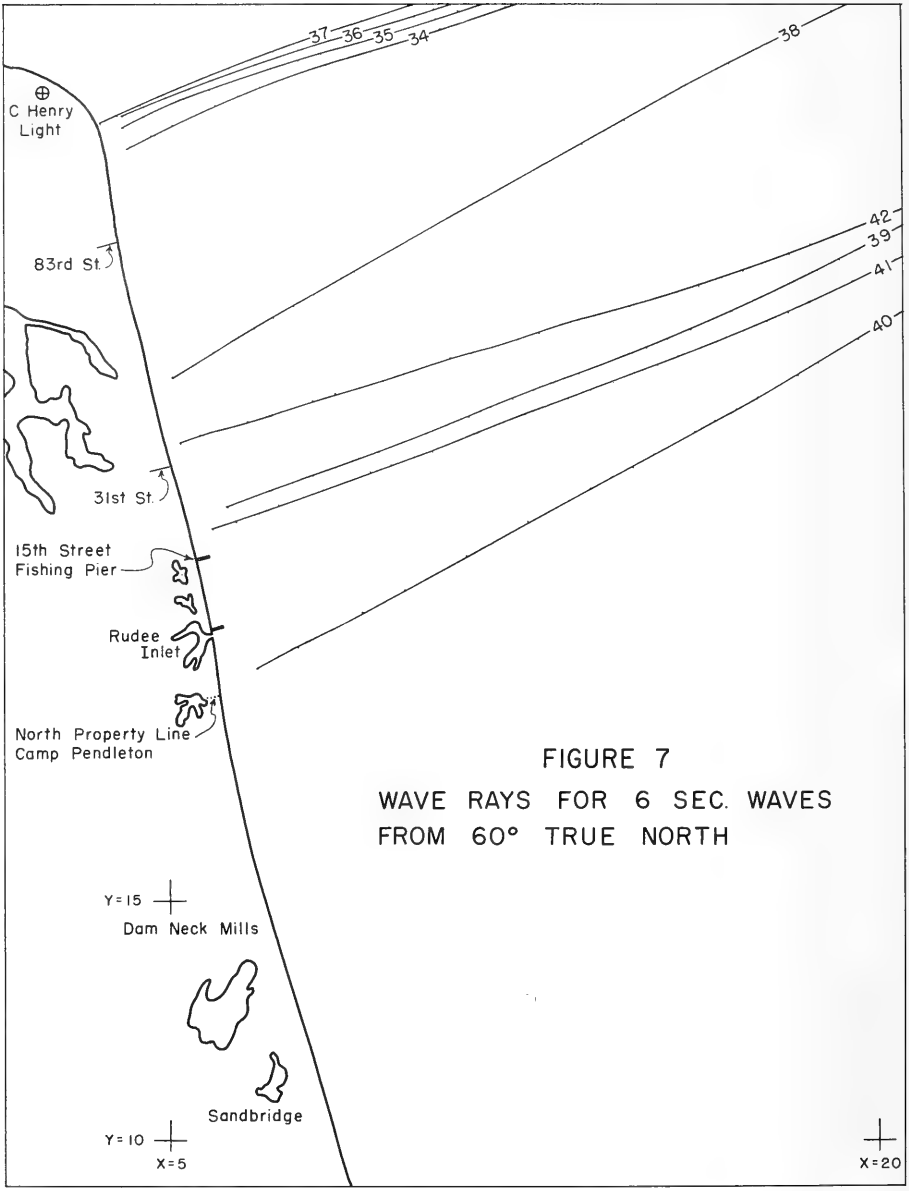


FIGURE 7
WAVE RAYS FOR 6 SEC. WAVES
FROM 60° TRUE NORTH

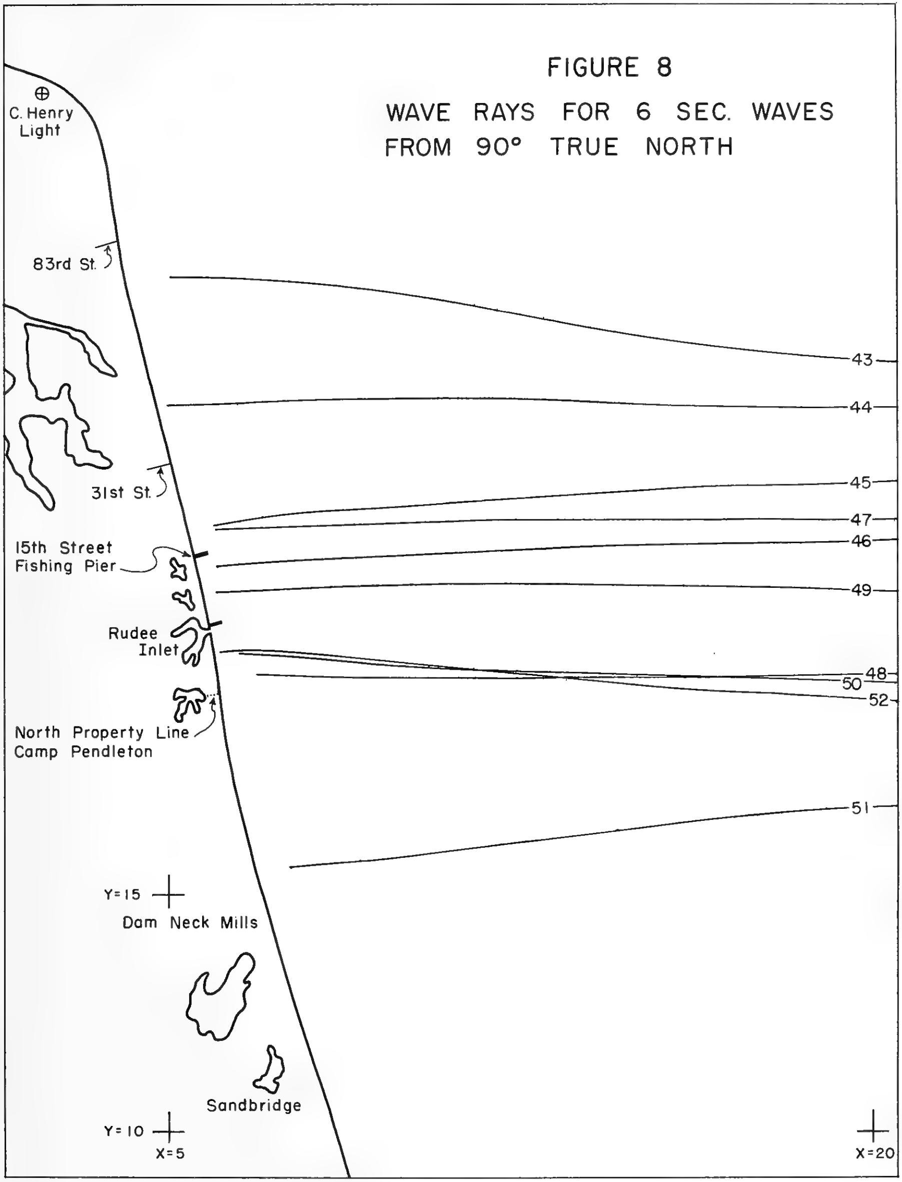
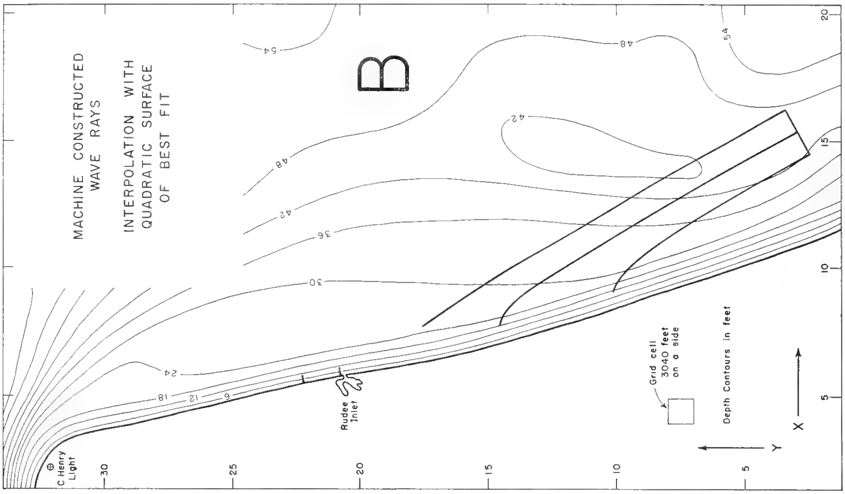
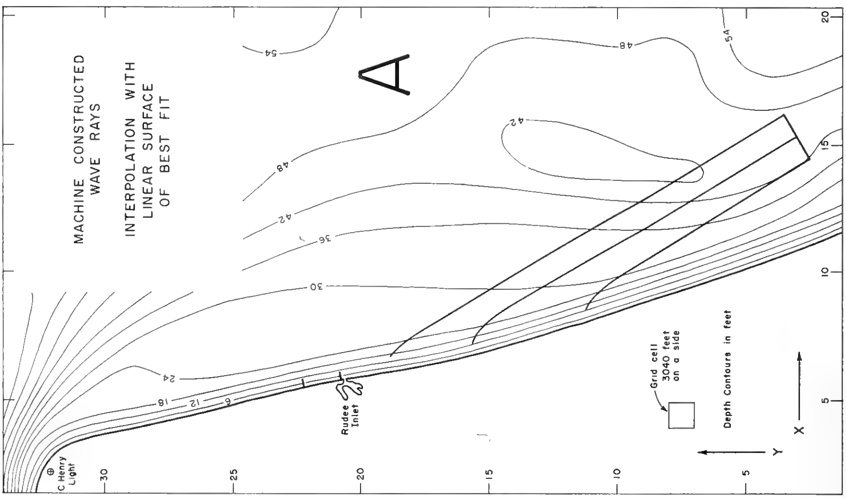


FIGURE 8
WAVE RAYS FOR 6 SEC. WAVES
FROM 90° TRUE NORTH
C. Henry
Light
83rd St.
31st St.
15th Street
Fishing Pier
Rudee
Inlet
North Property Line
Camp Pendleton
Y=15
Dam Neck Mills
Sandbridge
Y=10
X=5
X=20
43
44
45
47
46
49
48
50
52
51



MACHINE CONSTRUCTED WAVE RAYS
INTERPOLATION WITH QUADRATIC SURFACE OF BEST FIT
B
C Henry Light
Rudee Inlet
Grid cell 3040 feet on a side
Depth Contours in feet



MACHINE CONSTRUCTED WAVE RAYS
INTERPOLATION WITH LINEAR SURFACE OF BEST FIT
A
C Henry Light
Rudee Inlet
Grid cell 3040 feet on a side
Depth Contours in feet

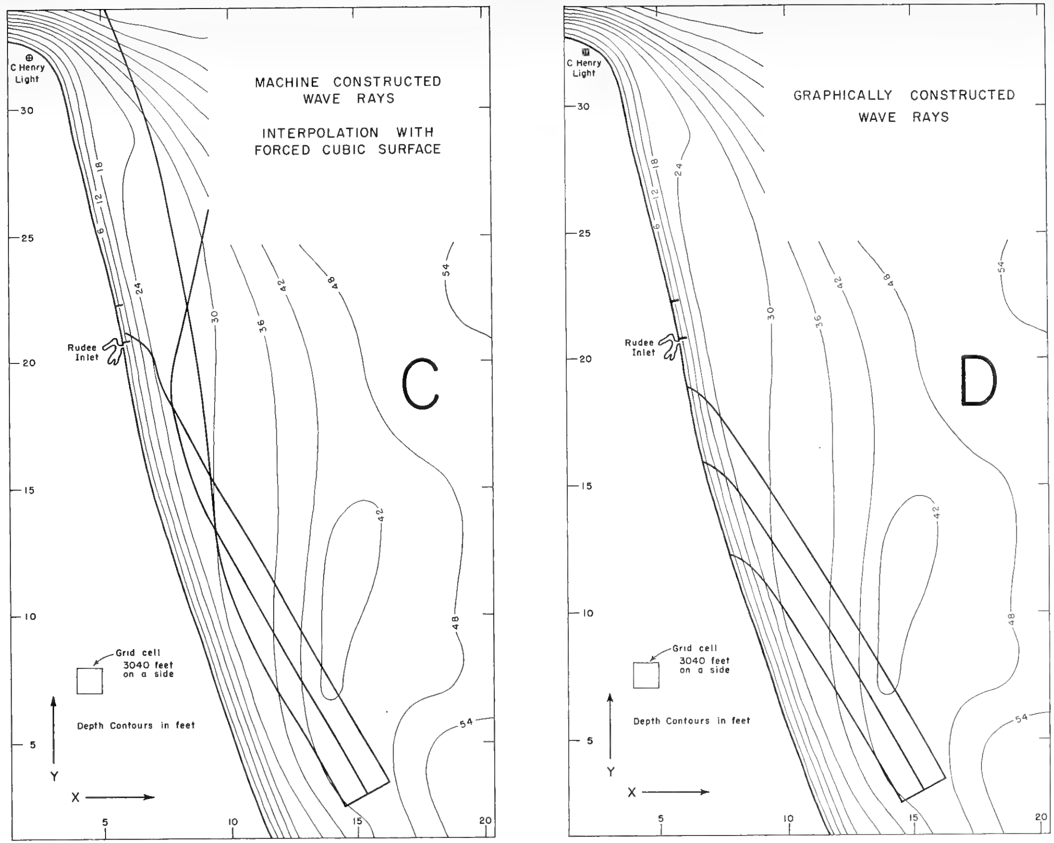


FIGURE 9. COMPARISONS OF WAVE RAYS 31-33 (Figure 2.) CONSTRUCTED BY THREE DIFFERENT PROGRAMS FOR FITTING VELOCITY SURFACES

APPENDIX A

Computer Program for Computation of Wave Velocities as a Function of Wave Period and Water Depth

PROGRAM TITLE: COMPV.

Variables Used in Program:

NOTT.......... Number of different wave periods for which velocity computations are to be made.

TT............ Wave period (seconds).

K,DEP......... Water depth (feet).

CXX(K)........ Wave velocity (feet/second) for water of K depth.

XL............ 1/2 deep-water wave length (feet) for a wave of TT period.

L............. XL rounded down to the nearest integer.

Summary of Program:

NOTT and the first TT are the first variables input to the computer. For water of K depth, where K ranges from 1 to L in one-foot increments, CXX's are then computed by an iteration procedure so that the third digit to the right of the decimal is significant. After all CXX's have been computed for the first TT, they are rounded to the nearest hundredth and output. Computations then proceed using the next TT. After computations have been made using NOTT different TT's, the program stops.

Remarks:

If wave periods greater than 20 seconds are to be used with this program, CXX will require larger dimensions.

The output from this program consists of a table of water depths and associated wave velocities for each specified wave period. The CXX values of these serve as input for the DISTV program in Appendix B.

The COMPV program, as well as several of the following programs, utilizes a subroutine for the internal rounding of values before they are output. This subroutine (ROUND) may profitably be described at this point.

SUBROUTINE TITLE: ROUND.

Variables Used in Subroutine:

VALUE......... Corresponds to the value in the calling program which is to be output.

DEC........... 10^N where N is the number of digits which are to be output to the right of the decimal.

Summary of Subroutine:

If there are N digits to the right of the decimal in the output specification for VALUE, ROUND tests the N+1st digit to see if it is equal to or greater than 5. If it is, the Nth digit is rounded up. Otherwise, the Nth digit remains unchanged.

Remarks:

Mode specifications, f and k, for ROUND must be idential with those for the calling program; k for ROUND must be one greater than the maximum number of digits to be output for any VALUE specified by the calling program.

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX SOURCE PROGRAM XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

```
* 8 8
C     1620, FORTRAN II, COMPUTATION OF WAVE VELOCITIES(FEET/SECOND) AS  COMPV 01
C     A FUNCTION OF WATER DEPTH(FEET) AND WAVE PERIOD(SECONDS).        COMPV 02
C     THEORY FROM H.O.PUB.234(1944), PROGRAMED BY W.S.WILSON,          COMPV 03
C     JUNE 17, 1964.                                                   COMPV 04
      DIMENSION CXX (1025)                                             COMPV 05
      TANHF(X) = (EXPF(X)-EXPF(-X))/(EXPF(X)+EXPF(-X))                 COMPV 06
      P = 3.1415927                                                    COMPV 07
      G = 32.2                                                         COMPV 08
      READ 10,NOTT                                                     COMPV 09
   10 FORMAT (I3)                                                      COMPV 10
      DO 1000 NOT=1,NOTT                                               COMPV 11
      READ 20,TT                                                       COMPV 12
   20 FORMAT (F5.1)                                                    COMPV 13
      XL = 0.5*G*(TT**2.0)/(2.0*P)                                     COMPV 14
      L = XL                                                           COMPV 15
      CXXO = TT*G/(2.0*P)                                              COMPV 16
      CCC = 5.5                                                        COMPV 17
      BAR = 2.0*P/TT                                                   COMPV 18
      DO 2000 K=1,L                                                    COMPV 19
      DEP = K                                                          COMPV 20
      DO 3000 M=1,90                                                   COMPV 21
      CXX(K) = CXXO*TANHF((BAR*DEP)/CCC)                               COMPV 22
      IF (ABSF(CXX(K)-CCC)-.0005) 5,3000,3000                          COMPV 23
 3000 CCC = (CXX(K)+CCC)/2.0                                           COMPV 24
    5 IF (SENSE SWITCH 1) 4,3                                          COMPV 25
    4 TYPE 900,K,M                                                     COMPV 26
  900 FORMAT (2HK=,I5,3H,M=,I3)                                        COMPV 27
    3 VALUE = CXX(K)                                                   COMPV 28
      CALL ROUND (VALUE,100.)                                          COMPV 29
 2000 CXX(K) = VALUE                                                   COMPV 30
      PUNCH 100, TT,L                                                  COMPV 31
  100 FORMAT (8HPERIOD =,F5.1,25H SECONDS, MAXIMUM DEPTH =,I5,7H. FEET.)COMPV 32
      PUNCH 200                                                        COMPV 33
  200 FORMAT (/5(5HDEPTH,1X,6HVELCTY,3X)/)                             COMPV 34
      PUNCH 300,(K,CXX(K),K=1,L)                                       COMPV 35
  300 FORMAT (5(I5,F7.2,3X))                                           COMPV 36
      PUNCH 700                                                        COMPV 37
  700 FORMAT (///)                                                     COMPV 38
 1000 CONTINUE                                                         COMPV 39
      TYPE 800                                                         COMPV 40
  800 FORMAT (16HTHIS IS THE END.)                                     COMPV 41
      END                                                              COMPV 42
* 8 8
      SUBROUTINE ROUND (VALUE,DEC)                                     ROUND 01
C     PROGRAMED BY W.S.WILSON, JULY 18, 1964.                          ROUND 02
      IVALUE = VALUE * DEC * 10.                                       ROUND 03
      IF (IVALUE) 100,104,100                                          ROUND 04
  104 VALUE = 0.0                                                      ROUND 05
```

```
      GO TO 103                                                        ROUND 06
  100 JVALUE = VALUE * DEC                                             ROUND 07
      XVALUE = IVALUE                                                  ROUND 08
      XVALUE = XVALUE / 10.                                            ROUND 09
      YVALUE = JVALUE                                                  ROUND 10
      IF ((XVALUE - YVALUE) - 0.5) 102,101,101                         ROUND 11
  101 VALUE  = (YVALUE + 1.) / DEC                                     ROUND 12
      GO TO 103                                                        ROUND 13
  102 VALUE  = YVALUE / DEC                                            ROUND 14
  103 RETURN                                                           ROUND 15
      END                                                              ROUND 16
```

```
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX  INPUT  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

 1                                                                        NOTT
 4.0                                                                        TT

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX  OUTPUT  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
```

PERIOD = 4.0 SECONDS, MAXIMUM DEPTH = 40. FEET.

DEPTH	VELCTY	DEPTH	VELCTY	DEPTH	VELCTY	DEPTH	VELCTY	DEPTH	VELCTY
1	5.60	2	7.82	3	9.45	4	10.77	5	11.87
6	12.83	7	13.67	8	14.40	9	15.06	10	15.65
11	16.17	12	16.64	13	17.07	14	17.45	15	17.79
16	18.10	17	18.37	18	18.62	19	18.84	20	19.04
21	19.22	22	19.37	23	19.51	24	19.64	25	19.75
26	19.84	27	19.93	28	20.00	29	20.07	30	20.12
31	20.17	32	20.22	33	20.26	34	20.29	35	20.32
36	20.34	37	20.36	38	20.38	39	20.40	40	20.41

APPENDIX B

Computer Program for Distribution of Wave Velocity Values Over a Grid of Depth Values as a Function of Wave Period

PROGRAM TITLE: DISTV.

Variables Used in Program:

TT............ Wave period (seconds).

I,J........... For a given X,Y position on a grid, I = X + 1, J = Y + 1, where the grid origin is at X = 0, Y = 0.

CMAT(I,J)..... For position I,J on the grid, CMAT represents, in the input, water depth (feet or fathoms). After conversion, CMAT represents, in the output, wave velocity (grid units/second).

MM,NN......... Maximum I and J, respectively, for the grid.

FMOP.......... Allows conversion of CMAT to feet, if input is in fathoms.

GRID.......... Grid interval (feet).

L............. 1/2 deep-water wave length (feet) rounded down to the nearest integer.

(CXX(K),K=1,L) For a wave of TT period, CXX is the array of wave velocities (feet/second) where K ranges from 1 to L in one-foot increments.

JX,JY......... On each output card JX and JY represents the X and Y grid coordinates of the first velocity value on that same card.

Summary of Program:

MM,NN,FMOP,GRID,TT, and (CXX(K),K=1,L) are the initial input. Then the first row of CMAT depth values is read into the computer. (Fathoms are converted if specified by FMOP.) For each depth value the computer "looks up" the associated CXX and divides this value by GRID. When all values in

the first row have been converted from a depth in feet to a velocity in grid units/second, they are rounded to the eighth place to the right of the decimal and output. When all rows for a given grid have been similarly input, converted, and output, the computer pauses in order that another set of data may be input.

Remarks:

Each output card includes (at the extreme right-hand side) TT, JX, and JY. Depths greater and L are assigned wave velocities equal to those assigned for L. The output from this program serves as input for MAIN 1620 and MAIN 7094 (Appendices D and E).

MM must be a multiple of 10.

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX SOURCE PROGRAM XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

```
* 8 8
C     1620, FORTRAN II, DISTRIBUTION OF WAVE VELOCITIES(GRID UNITS/       DISTV 01
C     SECOND) OVER A GRID OF DEPTHS(FEET OR FATHOMS) AS A FUNCTION        DISTV 02
C     OF WAVE PERIOD(SEC.),PROGRAMED BY W.S.WILSON, MAY 19,1964.          DISTV 03
      DIMENSION CXX(1025),CMAT(200)                                       DISTV 04
   10 READ 40, MM,NN,FMOP,GRID,TT                                         DISTV 05
   40 FORMAT (I4,I4,F2.0,F7.0,F5.1)                                       DISTV 06
      XL = 0.5*5.118*(TT**2.0)                                            DISTV 07
      L = XL                                                              DISTV 08
      READ 20, (CXX(K),K=1,L)                                             DISTV 09
   20 FORMAT (5(5X,F7.2,3X))                                              DISTV 10
      PUNCH 100,TT,GRID                                                   DISTV 11
  100 FORMAT (8HPERIOD =,F5.1,17H SEC.,GRID SIZE =,F7.0,6H FEET./)        DISTV 12
      DO 3000 J=1,NN                                                      DISTV 13
      READ 30, (CMAT(I),I=1,MM)                                           DISTV 14
   30 FORMAT (10F4.1)                                                     DISTV 15
      IF (FMOP) 2,2,1                                                     DISTV 16
    1 DO 1000 I=1,MM                                                      DISTV 17
 1000 CMAT(I) = 6.0*CMAT(I)                                               DISTV 18
    2 DO 2000 I=1,MM                                                      DISTV 19
      NVALUE = 1                                                          DISTV 20
      IF (CMAT(I)) 3,2000,4                                               DISTV 21
    3 CMAT(I) = -(CMAT(I))                                                DISTV 22
      NVALUE = 2                                                          DISTV 23
    4 IF (CMAT(I)-XL) 6,6,5                                               DISTV 24
    5 CMAT(I) = XL                                                        DISTV 25
    6 XK = CMAT(I)                                                        DISTV 26
      K = XK                                                              DISTV 27
      CMAT(I) = CXX(K)/GRID                                               DISTV 28
      CALL ROUND (CMAT(I),1.E8)                                           DISTV 29
      GO TO (2000,7),NVALUE                                               DISTV 30
    7 CMAT(I) = -(CMAT(I))                                                DISTV 31
 2000 CONTINUE                                                            DISTV 32
      MM5 = MM/5                                                          DISTV 33
      II = 1                                                              DISTV 34
      DO 5000 JM=1,MM5                                                    DISTV 35
      JX = II-1                                                           DISTV 36
      JY = J-1                                                            DISTV 37
      III = II+4                                                          DISTV 38
      PUNCH 200,(CMAT(I),I=II,III),TT,JX,JY                               DISTV 39
  200 FORMAT (5(F10.8,3X),2X,F5.1,I4,I4)                                  DISTV 40
      II = III+1                                                          DISTV 41
 5000 CONTINUE                                                            DISTV 42
 3000 CONTINUE                                                            DISTV 43
      TYPE 400,TT                                                         DISTV 44
  400 FORMAT (F5.1,20H SEC.GRID COMPLETED.)                               DISTV 45
      PAUSE                                                               DISTV 46
      PUNCH 300                                                           DISTV 47
```

```
300 FORMAT (///)                                                      DISTV 48
    GO TO 10                                                          DISTV 49
    END                                                               DISTV 50
```

```
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX  INPUT  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

 20   3 0  3040.  4.0                                      MM,NN,FMOP,GRID,TT
   1   5.60       2   7.82       3   9.45       4  10.77       5  11.87
   6  12.83       7  13.67       8  14.40       9  15.06      10  15.65
  11  16.17      12  16.64      13  17.07      14  17.45      15  17.79
  16  18.10      17  18.37      18  18.62      19  18.84      20  19.04
  21  19.22      22  19.37      23  19.51      24  19.64      25  19.75
  26  19.84      27  19.93      28  20.00      29  20.07      30  20.12
  31  20.17      32  20.22      33  20.26      34  20.29      35  20.32
  36  20.34      37  20.36      38  20.38      39  20.40      40  20.41
-00 -00 -00 -00 -00 -00 -00 -00 -00 -41                                     0000
-32 -24 -09 027 030 040 043 045 051 055                                     1000
-00 -00 -00 -00 -00 -00 -00 -00 -00 -43                                     0001
-32 -22 008 025 030 039 043 045 051 056                                     1001
-00 -00 -00 -00 -00 -00 -00 -00 -48 -43                                     0002
-30 -15 020 030 036 042 045 049 055 057                                     1002
```

```
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX  OUTPUT  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

PERIOD =  4.0 SEC.,GRID SIZE =  3040. FEET.

0.00000000   0.00000000   0.00000000   0.00000000   0.00000000       4.0   0   0
0.00000000   0.00000000   0.00000000   0.00000000   -.00671382       4.0   5   0
-.00665132   -.00646053   -.00495395    .00655592    .00661842       4.0  10   0
 .00671382    .00671382    .00671382    .00671382    .00671382       4.0  15   0
0.00000000   0.00000000   0.00000000   0.00000000   0.00000000       4.0   0   1
0.00000000   0.00000000   0.00000000   0.00000000   -.00671382       4.0   5   1
-.00665132   -.00637171    .00473684    .00649671    .00661842       4.0  10   1
 .00671053    .00671382    .00671382    .00671382    .00671382       4.0  15   1
0.00000000   0.00000000   0.00000000   0.00000000   0.00000000       4.0   0   2
0.00000000   0.00000000   0.00000000   -.00671382   -.00671382       4.0   5   2
-.00661842   -.00585197    .00626316    .00661842    .00669079       4.0  10   2
 .00671382    .00671382    .00671382    .00671382    .00671382       4.0  15   2
```

APPENDIX C

Computer Program to Produce Matrices for Deriving Equations of Linear Surfaces

PROGRAM TITLE: PRMAT.

Variables Used in Program:

(X(I),I=1,4).. The X co-ordinates of the four data points.

(Y(I),I=1,4).. The Y co-ordinates of the four data points.

S,EM......... Matrices used in determining the coefficients for an equation of a linear surface of best-fit to data values at four points.

Summary of Program:

Let C represent wave velocity at any grid position X,Y. By specifying the X,Y values for four positions, this program outputs S and EM. When later manipulated with a particular set of C values for these same four positions, S and EM produce the coefficients (E's) of linear equation of the form, $C = E_1 + E_2X + E_3Y$. This equation represents the least-squares plane of best-fit to this particular set of C values.

Remarks:

This program was extracted from a general program provided by W. C. Krumbein that fits a first, second, or third-order polynomial surface to specified regularly-spaced data points by the least-squares method.

The output from this program serves as input to MAIN 1620 and MAIN 7094 (Appendices D and E).

```
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX  SOURCE PROGRAM  XXXXXXXXXXXXXXXXXXXXXXXXXXXXX

*16 6
C     1620, FORTRAN II, PRODUCE MATRICES FOR DERIVING EQUATIONS OF
C     LINEAR SURFACES.
C     THEORY FROM W.C.KRUMBEIN, PROGRAMED BY BETTY BENSON,
C     MODIFIED BY W.S.WILSON, JULY 17, 1964.
      DIMENSION X(4), Y(4), EM(4,3), S(3,3)
      READ 10, (X(I),I=1,4)
      READ 10, (Y(I),I=1,4)
   10 FORMAT (4F5.0)
      DO 1000 L=1,4
      EM(L,1) = 1.
      EM(L,2) = X(L)
 1000 EM(L,3) = Y(L)
      DO 2000 I=1,3
      DO 2000 J=1,3
      S(I,J) = 0.0
      DO 3000 L=1,4
 3000 S(I,J) = S(I,J) +EM(L,I)*EM(L,J)
 2000 CONTINUE
      DO 4000 K=1,3
      DIV = S(K,K)
      S(K,K) = 1.0
      DO 5000 J=1,3
 5000 S(K,J) = S(K,J)/DIV
      DO 4000 I=1,3
      IF (I-K) 1,4000,1
    1 DIV = S(I,K)
      S(I,K) = 0.0
      DO 6000 J=1,3
 6000 S(I,J) = S(I,J)-DIV*S(K,J)
 4000 CONTINUE
      PUNCH 500
  500 FORMAT (50HMATRICES FOR DERIVING EQUATIONS OF LINEAR SURFACES)
      PUNCH 100, ((S(I,J),J=1,3),I=1,3)
  100 FORMAT (6F12.8)
      PUNCH 200, ((EM(L,I),L=1,4),I=1,3)
  200 FORMAT (12F6.2)
      END

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX  INPUT  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

  0.0  1.0  1.0  0.0                                              (X(I),I=1,4)
  0.0  0.0  1.0  1.0                                              (Y(I),I=1,4)

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX  OUTPUT  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

MATRICES FOR DERIVING EQUATIONS OF LINEAR SURFACES
   .75000000  -.50000000  -.50000000  -.50000000  1.00000000  0.00000000
  -.50000000  0.00000000  1.00000000
  1.00  1.00  1.00  1.00  0.00  1.00  1.00  0.00  0.00  0.00  1.00  1.00
```

APPENDIX D

Computer Program for Wave Refraction (Linear-Interpolation) Using the IBM 1620

PROGRAM TITLE: MAIN 1620.

Input Variables:

S,EM......... Matrices used in determining coefficients for the plane of best-fit to four data points.

XLABL........ Arbitrary title used to designate each batch of input.

MM,NN......... Maximum values of I and J, respectively, for the CMAT grid of velocity values; I = X + 1, J = Y + 1, with the grid origin located at X = 0, Y = 0.

CHECK......... Allows either a two-dimensional CMAT to be input, or a single-rowed CMAT (extending from shore to deep water) to be input and all other rows to be made identical to the first. (This procedure creates a set of grid values which can be characterized by straight and parallel contours.)

TT............ Wave period (seconds).

NOJ........... Number of wave rays to be run in each batch.

D............. Distance incremented between successive points along a ray path.

CMAT(I,J)..... Wave velocity (grid units/second) at grid position I,J.

A............. Angle (degrees) measured from the direction of increasing X along the X-axis moving counter-clockwise to the direction of travel of a wave ray.

X,Y........... Grid origin position for a wave ray.

Output Variables:

XLABL......... Defined previously.

TT............ Defined previously.

NOT........... Batch number.

N............. Ray number for batch NOT.

MAX........... Number assigned to a point along a ray path where calculations are made. MAX = 1 at the origin point of ray.

XXX,YYY....... X,Y co-ordinates for a given MAX.

ANGLE......... Angle (degrees) between X-axis and ray, as previously defined for A.

PCTDIF........ Percent difference between the value of one of the four points to which a plane is fit and that corresponding value of the plane (see text, "Interpolation Surfaces").

Variables in Common:

S, EM......... Defined previously.

E............. Coefficients of the equation of a plane of best-fit to four grid points.

YVW........... Matrix used in determining E's.

CMAT.......... Defined previously.

C............. Values of the four CMAT values to which a plane is fit.

XLABL......... Defined previously.

D............. Defined previously.

TT............ Defined previously.

CXY........... Wave-velocity for a given MAX.

IT............ Number of iterations used in obtaining the curvature of a wave ray for a given MAX.

NGO........... Designates whether a wave ray has moved within one grid unit

of the edge of the grid.

AMM,ANN....... Used in determining NGO.

MAX........... Defined previously.

MIT........... Designates whether the last two curvature estimates for a given MAX are less than 0.00009/D, whether the 18th and the 20th estimates are less than this value, or whether neither of the above is true.

Summary of Program:

MAIN reads S and EM and sets NOT = 1 to denote batch one. It then reads the information (XLABL, MM, NN, CHECK, TT, NOT, D, and CMAT) for processing the first batch of rays. For N = 1 the program reads the information (X, Y, and A) for processing the first wave ray. MAIN converts A from degrees to radians and then punches the title information (XLABL, TT, NOT, N, and the column headings). Control is then transferred to the RAYN subroutine. When RAYN has determined the path of the first ray, control is transferred back to MAIN. MAIN then reads the information for the second ray (N = 2) and proceeds as before. When NOJ ray paths have been determined for NOT = 1, the computer pauses allowing time to load the necessary information for the second batch. After pressing start, MAIN sets NOT = 2, and the information for this batch is read. MAIN then continues as before. The program is terminated when the desired number of batches has been processed.

Remarks:

This program calls four subroutines, RAYN, SURFCE, MOVE, and ROUND. Descriptions of these subroutines follow. (ROUND has been described previously.)

SUBROUTINE TITLE: RAYN.

Summary of Subroutine:

RAYN calls SURFCE to obtain FK (ray curvature) for MAX = 1. MAX is then set equal to two. RAYN calls MOVE in order that X, Y, A, and FK be obtained for MAX = 2. ANGLE (equal to A but expressed in degrees instead of radians) and PCTDIF are then computed. XXX and YYY are set equal to X and Y, respectively, so that significance is not lost when the values to be output are rounded. ROUND is called for XXX, YYY, ANGLE, and PCTDIF before they are punched. MAX is incremented by one, and the entire process continues until 1) MAX = 500, 2) CXY is equal to or less than zero, 3) MIT = 3, or 4) NGO = 3. The first condition guards against a ray going in an endless circle. The second prevents a ray from leaving the water and going over land. The third prevents invalid curvature approximations. The fourth condition stops a ray if it reaches the edge of the grid. Any of these four conditions causes the message, "RAY STOPPED" to be punched and transfers control back to MAIN. If MIT = 2, the message "CURVATURE APPROXIMATED" is punched and computations proceed as usual.

Remarks:

The use of Sense Switch 3 keeps the operator informed of the current value of MAX.

SUBROUTINE TITLE: SURFCE.

Variables Used in Subroutine:

I,FI.......... X + 1 rounded down to the nearest integer.

J,FJ.......... Y + 1 rounded down to the nearest integer.

XL............ X + 1 - FI.

YL............ Y + 1 - FJ.

ZI,ZJ......... Values of FI and FJ, respectively, the last time SURFCE was called. (Not valid for MAX = 1.)

FK............ Curvature of the wave ray at X,Y.

Summary of Subroutine:

For a specified A and X,Y position on the CMAT grid, SURFCE first determines I, J, XL, and YL. If MAX does not equal one, SURFCE checks to see if ZI = FI and ZJ = FJ. If both equalities are true, the E values from the previous operation of SURFCE are still valid, and the CXY and FK computations can be made directly. Otherwise, it is necessary to derive new E values, using C, EM, and S, before computing CXY and FK.

Remarks:

Program steps corresponding to SURFCE20 through SURCE27 in the listing that follows were obtained from a program provided by Dr. W. C. Krumbein. (This program is described with the PRMAT program in Appendix C.)

SUBROUTINE TITLE: MOVE.

Variables Used in Subroutine:

FKBAR......... Curvature used to obtain DELA for a given IT.

FKKPP......... FKBAR for IT = 18.

FKKP.......... FKBAR for IT = n-1 where current FKBAR is for IT = n. (For IT = 1, FKKP equals the FKBAR used in determining XX and YY the previous time MOVE was called.)

XX, YY, AA,

FKK........... X, Y, A, and FK values, respectively, for MAX = n + 1 where MAX = n when MOVE was called.

DELX.......... XX - X.

DELY.......... YY - Y.

DELA.......... AA - A.

ABAR.......... (A + AA)/2.

Summary of Subroutine:

If MAX equals three or more when MOVE is called, FKBAR for the previous MAX is used in obtaining approximations of XX, YY, and AA. (For MAX = 2, FKBAR is set equal to FK.) SURFCE is called and returns FKK for this approximation at XX,YY. FKBAR is then redefined as (FK+FKK)/2. If the difference between FKBAR and FKKP is less than 0.00009/D, the current XX, YY, AA, and FKK values are accepted for the new point. If the difference is greater, FKKP is set equal to FKBAR, and the current FKBAR is used to obtain another set of XX, YY, AA, and FKK values. The difference between FKBAR and FKKP is again tested. This cycle may repeat a maximum of 20 times before termination. If the cycle stops before IT = 20, MIT is set equal to one. If the cycle stops at IT = 20, and if the difference between FKBAR and FKKPP is less than 0.00009/D, then MIT is set equal to two, and FKBAR is defined as (FKBAR+FKKP)/2 for obtaining XX, YY, AAA, and FKK. If IT = 20, and this difference is greater than 0.00009/D, then MIT is set equal to three. When MIT = 3, control is transferred back to RAYN immediately. When MIT = 1 or 2, XX and YY are tested to see if the new point has reached the edge of the grid. NGO = 2 if the ray has reached the edge of the grid, and NGO = 1 if it has not.

Remarks:

MIT = 1 when the curvature approximations are converging to a single value. MIT = 2 when the approximations are oscillating between two values. In this case the average of the two values is taken as the new curvature. MIT = 3 when the approximations are oscillating among three or more values. No valid curvature approximation can be made in this case; this is one basis for the terminiation of a ray.

The use of Sense Switch 2 allows the operator to observe successive IT and

FKBAR values.

The maximum difference (0.00009/D) in the curvature approximations is such that the output ANGLE is significant in the hundredths digit.

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX SOURCE PROGRAM XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

```
* 8 6
C      1620, FORTRAN II, LINEAR-INTERPOLATION WAVE REFRACTION PROGRAM.
C      ORIGINAL PROGRAM BY GRISWOLD,NAGLE,AND MEHR. THIS PROGRAM ADAPTED
C      FROM ORIGINAL BY WILSON,HARRISON,KRUMBEIN,AND BENSON. 8/4/64.
       DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(50,51),C(4),XLABL(12)
       COMMON S,EM,E,YVW,CMAT,C,XLABL,D,TT,CXY,IT,NGO,AMM,ANN,MAX,MIT
       READ 5,((S(I,J),J=1,3),I=1,3)
     5 FORMAT(6F12.8)
       READ 7,((EM(L,I),L=1,4),I=1,3)
     7 FORMAT(12F6.2)
       NOT = 1
  9998 READ 400, XLABL
   400 FORMAT( 12A4)
       READ 402,MM,NN,CHECK,TT,NOJ,D
   402 FORMAT (2I4,F3.0,7X,F5.1,I5,F4.1)
       AMM = MM-1
       ANN = NN-1
       IF(CHECK) 10,10,20
    10 READ 11,((CMAT(I,J),I=1,MM),J=1,NN)
    11 FORMAT (5(F10.8,3X))
       GO TO 14
    20 J = 1
       READ 11, (CMAT(I,J),I=1,MM)
       DO 77 J=2,NN
       DO 77 I=1,MM
    77 CMAT(I,J) = CMAT(I,1)
    14 DO 15 N=1, NOJ
       READ 6,A,X,Y
     6 FORMAT (F7.2,2F6.2)
       MAX = 1
       PUNCH 403,XLABL,TT,NOT,N,MAX,X,Y,A
   403 FORMAT (///12A4/8HPERIOD =,F5.1,6H SEC.,,10H BATCH NO.,I3,9H, RAY
      1NO.,  I3,1H.//4X,3HMAX,6X,1HX,8X,1HY,8X,5HANGLE,4X,6HPCTDIF,
      2//I7,2F9.2,F11.2)
       A=A*.0174532925
       CALL RAYN (X,Y,A)
    15 CONTINUE
       PAUSE
       NOT = NOT + 1
       GO TO 9998
       END
* 8 6
       SUBROUTINE RAYN (X,Y,A)
C
       DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(50,51),C(4),XLABL(12)
       COMMON S,EM,E,YVW,CMAT,C,XLABL,D,TT,CXY,IT,NGO,AMM,ANN,MAX,MIT
       CALL SURFCE (X,Y,A,FK)
     3 MAX=1+MAX
```

```
      IF (SENSE SWITCH 3) 100,101                                       RAYN  07
  100 TYPE 103,MAX                                                      RAYN  08
  103 FORMAT (4HMAX=,I4)                                                RAYN  09
  101 IF (MAX-500) 399,15,15                                            RAYN  10
  399 CALL MOVE (X,Y,A,FK)                                              RAYN  11
      IF (CXY) 15,15,396                                                RAYN  12
  396 GO TO (397,395,15), MIT                                           RAYN  13
  395 WRITE 200, MAX                                                    RAYN  14
  200 FORMAT (32HCURVATURE APPROXIMATED FOR MAX =,I4)                   RAYN  15
  397 ANGLE=A*57.29577951                                               RAYN  16
      XXX = X                                                           RAYN  17
      YYY = Y                                                           RAYN  18
      PCTDIF = ABSF((C(3)-E(1)-E(2)-E(3))/C(3))*100.                    RAYN  19
      CALL ROUND (XXX,100.)                                             RAYN  20
      CALL ROUND (YYY,100.)                                             RAYN  21
      CALL ROUND (ANGLE,100.)                                           RAYN  22
      CALL ROUND (PCTDIF,10.)                                           RAYN  23
      PUNCH 12,MAX,XXX,YYY,ANGLE,PCTDIF                                 RAYN  24
   12 FORMAT (I7,2F9.2,F11.2,F10.1)                                     RAYN  25
      GO TO (3,15),NGO                                                  RAYN  26
   15 PUNCH 13                                                          RAYN  27
   13 FORMAT (12HRAY STOPPED.)                                          RAYN  28
      RETURN                                                            RAYN  29
      END                                                               RAYN  30
* 8 6
      SUBROUTINE SURFCE (X,Y,A,FK)                                      SURFCE01
C                                                                       SURFCE02
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(50,51),C(4),XLABL(12)   SURFCE03
      COMMON S,EM,E,YVW,CMAT,C,XLABL,D,TT,CXY,IT,NGO,AMM,ANN,MAX,MIT    SURFCE04
      I=X+1.                                                            SURFCE05
      J=Y+1.                                                            SURFCE06
      FI=I                                                              SURFCE07
      FJ=J                                                              SURFCE08
      XL=X+1.-FI                                                        SURFCE09
      YL=Y+1.-FJ                                                        SURFCE10
      IF (MAX-1) 1,1,4                                                  SURFCE11
    4 IF (ZI-FI) 1,2,1                                                  SURFCE12
    2 IF (ZJ-FJ) 1,3,1                                                  SURFCE13
    1 ZI = FI                                                           SURFCE14
      ZJ = FJ                                                           SURFCE15
      C(1)=CMAT(I,J)                                                    SURFCE16
      C(2)=CMAT(I+1,J)                                                  SURFCE17
      C(3)=CMAT(I+1,J+1)                                                SURFCE18
      C(4)=CMAT(I,J+1)                                                  SURFCE19
      DO 318 II=1,3                                                     SURFCE20
      YVW(II) = 0.                                                      SURFCE21
      DO 318 L=1,4                                                      SURFCE22
  318 YVW(II) = YVW(II)+C(L)*EM(L,II)                                   SURFCE23
      DO 319 II=1,3                                                     SURFCE24
      E(II) = 0.                                                        SURFCE25
```

```
      DO 319 JJ=1,3
  319 E(II) = E(II)+S(II,JJ)*YVW(JJ)
    3 CXY = E(1) + E(2)*XL + E(3)*YL
      FK =(E(2)*SINF(A)-E(3)*COSF(A))/CXY
      RETURN
      END
* 8 6
      SUBROUTINE MOVE (X,Y,A,FK)
C
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(50,51),C(4),XLABL(12)
      COMMON S,EM,E,YVW,CMAT,C,XLABL,D,TT,CXY,IT,NGO,AMM,ANN,MAX,MIT
      IF (MAX - 2) 102,102,104
  102 FKBAR=FK
  104 MIT = 1
      DO 20 IT=1,20
   39 DELA=FKBAR*D
      AA=A+DELA
      ABAR=A+.5*DELA
      DELX=D*COSF(ABAR)
      DELY=D*SINF(ABAR)
      XX=X+DELX
      YY=Y+DELY
      GO TO (101,6), MIT
  101 CALL SURFCE (XX,YY,AA,FKK)
      IF (CXY) 38,38,10
   10 FKBAR = 0.5 * (FK + FKK)
      IF (SENSE SWITCH 2) 898,899
  898 TYPE 900, IT,FKBAR
  900 FORMAT (I4,E14.8)
  899 IF (IT - 18) 5,37,9
   37 FKKPP = FKBAR
    5 IF (MAX - 2) 7,7,9
    7 IF (IT - 1) 20,20,9
    9 IF (ABSF(FKKP-FKBAR) - (0.00009/D)) 6,6,20
   20 FKKP = FKBAR
      IF (ABSF(FKKPP - FKBAR) - (0.00009/D)) 18,18,17
   17 MIT = 3
      GO TO 38
   18 FKBAR = 0.5 * (FKBAR + FKKP)
      MIT = 2
      GO TO 39
    6 NGO = 1
      IF ((XX-1.0)*((AMM-1.0)-XX))2,2,3
    3 IF ((YY-1.0)*((ANN-1.0)-YY))2,2,8
    2 NGO = 2
    8 X = XX
      Y = YY
      A = AA
      FK = FKK
   38 RETURN
      END
```

```
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX  INPUT  XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

   .75000000  -.50000000  -.50000000  -.50000000  1.00000000  0.00000000       L1
  -.50000000  0.00000000  1.00000000                                           L2
  1.00  1.00  1.00  1.00  0.00  1.00  1.00  0.00  0.00  0.00  1.00  1.00       L3
1620, GRID OFF VA.CAPES, LINEAR INTERP., 8/5/64.                            XLABL
  20  22  0         4.0     3  .5                                         BATCH 1
0.00000000   0.00000000   0.00000000   0.00000000   0.00000000      4.0   0    0
0.00000000   0.00000000   0.00000000   0.00000000   -.00671382      4.0   5    0
-.00665132   -.00646053   -.00495395    .00655592    .00661842      4.0  10    0
 .00671382    .00671382    .00671382    .00671382    .00671382      4.0  15    0
0.00000000   0.00000000   0.00000000   0.00000000   0.00000000      4.0   0    1
0.00000000   0.00000000   0.00000000   0.00000000   -.00671382      4.0   5    1
-.00665132   -.00637171    .00473684    .00649671    .00661842      4.0  10    1
 .00671053    .00671382    .00671382    .00671382    .00671382      4.0  15    1
0.00000000   0.00000000   0.00000000   0.00000000   0.00000000      4.0   0    2
0.00000000   0.00000000   0.00000000   -.00671382   -.00671382      4.0   5    2
-.00661842   -.00585197    .00626316    .00661842    .00669079      4.0  10    2
 .00671382    .00671382    .00671382    .00671382    .00671382      4.0  15    2
0.00000000   0.00000000   0.00000000   0.00000000   0.00000000      4.0   0    3
0.00000000   0.00000000   -.00671382   -.00671382   -.00669079      4.0   5    3
-.00646053    .00354276    .00657895    .00669079    .00671382      4.0  10    3
 .00671382    .00671382    .00671382    .00671382    .00671382      4.0  15    3
0.00000000   0.00000000   0.00000000   0.00000000   0.00000000      4.0   0    4
0.00000000   0.00000000   -.00671382   -.00670395   -.00666447      4.0   5    4
-.00612500    .00514803    .00665132    .00669737    .00671382      4.0  10    4
 .00671382    .00671382    .00671382    .00671382    .00671382      4.0  15    4
0.00000000   0.00000000   0.00000000   0.00000000   0.00000000      4.0   0    5
0.00000000   0.00000000   -.00671382   -.00670395   -.00661842      4.0   5    5
-.00495395    .00637171    .00661842    .00671382    .00671382      4.0  10    5
 .00671382    .00671382    .00671382    .00671382    .00671382      4.0  15    5
0.00000000   0.00000000   0.00000000   0.00000000   0.00000000      4.0   0    6
0.00000000   -.00671382   -.00670395   -.00666447   -.00649671      4.0   5    6
 .00422039    .00660197    .00669079    .00671382    .00671382      4.0  10    6
 .00671382    .00671382    .00671382    .00671382    .00671382      4.0  15    6
0.00000000   0.00000000   0.00000000   0.00000000   0.00000000      4.0   0    7
0.00000000   -.00669737   -.00669737   -.00666447   -.00612500      4.0   5    7
 .00531908    .00666447    .00670395    .00671382    .00671382      4.0  10    7
 .00671382    .00671382    .00671382    .00671382    .00671382      4.0  15    7
0.00000000   0.00000000   0.00000000   0.00000000   0.00000000      4.0   0    8
0.00000000   -.00669737   -.00668421   -.00665132   -.00495395      4.0   5    8
 .00655592    .00666447    .00669737    .00671382    .00671382      4.0  10    8
 .00671382    .00671382    .00671382    .00671382    .00671382      4.0  15    8
0.00000000   0.00000000   0.00000000   0.00000000   0.00000000      4.0   0    9
-.00669737   -.00666447   -.00665132   -.00652632    .00390461      4.0   5    9
 .00663487    .00668421    .00669737    .00671382    .00671382      4.0  10    9
 .00671382    .00671382    .00671382    .00671382    .00671382      4.0  15    9
0.00000000   0.00000000   0.00000000   0.00000000   0.00000000      4.0   0   10
-.00669079   -.00666447   -.00665132   -.00619737    .00531908      4.0   5   10
 .00665132    .00668421    .00670395    .00671382    .00671382      4.0  10   10
 .00668421    .00671382    .00671382    .00671382    .00671382      4.0  15   10
```

0.00000000	0.00000000	0.00000000	0.00000000	0.00000000	4.0	0	11
-.00667434	-.00666447	-.00652632	-.00547368	.00649671	4.0	5	11
.00665132	.00666447	.00669737	.00671382	.00669079	4.0	10	11
.00671382	.00671382	.00671382	.00671382	.00671382	4.0	15	11
0.00000000	0.00000000	0.00000000	0.00000000	-.00669079	4.0	0	12
-.00666447	-.00665132	-.00649671	.00257237	.00657895	4.0	5	12
.00665132	.00667434	.00671053	.00671382	.00669079	4.0	10	12
.00669737	.00671382	.00671382	.00671382	.00671382	4.0	15	12
0.00000000	0.00000000	0.00000000	0.00000000	-.00666447	4.0	0	13
-.00665132	-.00661842	-.00626316	.00514803	.00661842	4.0	5	13
.00667434	.00666447	.00669079	.00671382	.00669737	4.0	10	13
.00669737	.00671053	.00671382	.00671382	.00671382	4.0	15	13
0.00000000	0.00000000	0.00000000	0.00000000	-.00669737	4.0	0	14
-.00655592	-.00641776	-.00585197	.00637171	.00661842	4.0	5	14
.00665132	.00667434	.00670395	.00671382	.00671382	4.0	10	14
.00669079	.00671053	.00671382	.00671382	.00671382	4.0	15	14
0.00000000	0.00000000	0.00000000	-.00668421	-.00661842	4.0	0	15
-.00649671	-.00637171	-.00257237	.00646053	.00657895	4.0	5	15
.00665132	.00667434	.00669079	.00671053	.00671382	4.0	10	15
.00671382	.00671382	.00671382	.00671382	.00671382	4.0	15	15
0.00000000	0.00000000	0.00000000	-.00665132	-.00657895	4.0	0	16
-.00646053	-.00626316	.00514803	.00655592	.00657895	4.0	5	16
.00661842	.00667434	.00669079	.00671382	.00671382	4.0	10	16
.00671382	.00671382	.00671382	.00671382	.00671382	4.0	15	16
0.00000000	0.00000000	0.00000000	-.00663487	-.00657895	4.0	0	17
-.00649671	-.00626316	.00547368	.00660197	.00660197	4.0	5	17
.00665132	.00669079	.00670395	.00671382	.00671382	4.0	10	17
.00671382	.00671382	.00671382	.00671382	.00671382	4.0	15	17
0.00000000	0.00000000	0.00000000	-.00660197	-.00657895	4.0	0	18
-.00649671	-.00585197	.00637171	.00652632	.00661842	4.0	5	18
.00667434	.00668421	.00669737	.00670395	.00671382	4.0	10	18
.00671382	.00671382	.00671382	.00671382	.00671382	4.0	15	18
0.00000000	0.00000000	0.00000000	-.00657895	-.00655592	4.0	0	19
-.00649671	-.00422039	.00649671	.00652632	.00660197	4.0	5	19
.00665132	.00666447	.00671053	.00671382	.00671382	4.0	10	19
.00671382	.00671382	.00671382	.00671382	.00671382	4.0	15	19
0.00000000	0.00000000	-.00663487	-.00655592	-.00652632	4.0	0	20
-.00641776	0.00000000	.00655592	.00655592	.00655592	4.0	5	20
.00663487	.00665132	.00669079	.00671382	.00671382	4.0	10	20
.00671382	.00671382	.00671382	.00671382	.00671382	4.0	15	20
0.00000000	0.00000000	-.00660197	-.00655592	-.00649671	4.0	0	21
-.00637171	.00547368	.00655592	.00655592	.00655592	4.0	5	21
.00665132	.00667434	.00670395	.00671382	.00671382	4.0	10	21
.00671382	.00671382	.00671382	.00671382	.00671382	4.0	15	21

120.0 14.50 02.50 RAY 1
120.0 15.36 02.99 RAY 2
120.0 16.22 03.49 RAY 3

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX OUTPUT XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

1620, GRID OFF VA.CAPES, LINEAR INTERP.,8/16/64.
PERIOD = 4.0 SEC., BATCH NO. 1, RAY NO. 1.

MAX	X	Y	ANGLE	PCTDIF
1	14.50	2.50	120.00	
2	14.25	2.93	120.07	.1
3	14.00	3.37	120.14	0.0
4	13.75	3.80	120.22	0.0
5	13.50	4.23	120.28	.1
6	13.24	4.66	120.33	.1
7	12.99	5.09	120.51	.3
8	12.74	5.52	120.80	.3
9	12.48	5.95	121.10	.3
10	12.22	6.38	121.28	0.0
11	11.96	6.81	121.48	.2
12	11.70	7.23	121.70	0.0
13	11.43	7.66	121.83	0.0
14	11.17	8.08	121.94	.1
15	10.90	8.51	122.19	.2
16	10.64	8.93	122.59	.2
17	10.37	9.35	122.88	.1
18	10.09	9.77	123.05	.1
19	9.81	10.18	125.27	4.4
20	9.51	10.58	129.56	4.4
21	9.18	10.95	133.85	4.4
22	8.75	11.22	162.47	30.3

RAY STOPPED.

1620, GRID OFF VA.CAPES, LINEAR INTERP.,8/16/64.
PERIOD = 4.0 SEC., BATCH NO. 1, RAY NO. 2.

MAX	X	Y	ANGLE	PCTDIF
1	15.36	2.99	120.00	
2	15.11	3.42	120.00	0.0
3	14.86	3.86	120.00	0.0
4	14.61	4.29	120.00	0.0
5	14.36	4.72	120.00	0.0
6	14.11	5.16	120.00	0.0
7	13.86	5.59	120.00	0.0
8	13.61	6.02	120.00	0.0
9	13.36	6.45	120.00	0.0
10	13.11	6.89	120.00	0.0
11	12.86	7.32	120.02	0.0
12	12.61	7.75	120.06	0.0
13	12.36	8.19	120.11	0.0

14	12.11	8.62	120.17	0.0
15	11.86	9.05	120.24	0.0
16	11.60	9.48	120.30	0.0
17	11.35	9.91	120.37	0.0
18	11.10	10.34	120.44	0.0
19	10.85	10.78	120.51	.1
20	10.59	11.21	120.57	0.0
21	10.34	11.64	120.65	0.0
22	10.08	12.07	120.71	.1
23	9.83	12.50	120.88	.1
24	9.57	12.92	121.19	.1
25	9.31	13.35	121.41	.1
26	9.05	13.78	121.55	.1
27	8.78	14.20	121.98	.5
28	8.52	14.63	122.72	.5
29	8.24	15.04	123.27	.4
30	7.91	15.42	139.33	29.1
31	7.46	15.63	171.13	29.1

RAY STOPPED.

1620, GRID OFF VA.CAPES, LINEAR INTERP.,8/16/64.
PERIOD = 4.0 SEC., BATCH NO. 1, RAY NO. 3.

MAX	X	Y	ANGLE	PCTDIF
1	16.22	3.49	120.00	
2	15.97	3.92	120.00	0.0
3	15.72	4.36	120.00	0.0
4	15.47	4.79	120.00	0.0
5	15.22	5.22	120.00	0.0
6	14.97	5.66	120.00	0.0
7	14.72	6.09	120.00	0.0
8	14.47	6.52	120.00	0.0
9	14.22	6.95	120.00	0.0
10	13.97	7.39	120.00	0.0
11	13.72	7.82	120.00	0.0
12	13.47	8.25	120.00	0.0
13	13.22	8.69	120.00	0.0
14	12.97	9.12	120.03	0.0
15	12.72	9.55	120.08	0.0
16	12.47	9.98	120.14	0.0
17	12.22	10.42	120.19	0.0
18	11.97	10.85	120.24	0.0
19	11.71	11.28	120.35	0.0
20	11.46	11.71	120.51	0.0
21	11.21	12.14	120.62	0.0
22	10.95	12.57	120.68	.1
23	10.70	13.00	120.71	.1
24	10.44	13.43	120.72	.1

25	10.19	13.86	120.73	.1
26	9.93	14.29	120.81	.1
27	9.67	14.72	120.96	.1
28	9.41	15.15	121.12	.1
29	9.16	15.58	121.29	.1
30	8.90	16.00	121.44	.1
31	8.63	16.43	121.56	.1
32	8.37	16.86	121.68	.1
33	8.11	17.28	121.79	.3
34	7.84	17.70	123.51	3.7
35	7.56	18.11	125.42	.5
36	7.26	18.52	125.92	.5
37	6.91	18.87	144.59	5.8

RAY STOPPED.

APPENDIX E

Computer Program for Wave Refraction (Linear-Interpolation) Using the IBM 7094

PROGRAM TITLE: MAIN 7094.

Summary of Program:

This program is essentially the same as that of MAIN 1620; both programs give identical results if processing the same input data. The chief differences between the programs are as follows:

(1) MAIN 7094 is written in FORTRAN IV, while MAIN 1620 is written in FORTRAN II.

(2) NOTT (the maximum number of batches to be run) is specified for MAIN 7094.

(3) Sense Switches are not used in MAIN 7094.

(4) ROUND subroutine is not used in MAIN 7094 since the 7094 computer automatically rounds off output data.

(5) Due to the large storage capacity of the 7094, dimensions for CMAT in MAIN 7094 may greatly exceed those in MAIN 1620.

(6) Since the 7094 computer outputs to a printer, MAIN 7094 has been programmed to print title information at the top of each output page.

(7) MAIN 7094 operates approximately 200 times faster than MAIN 1620.

Remarks:

MAIN 1620 is best suited for experimental work connected with further program refinements and development, while MAIN 7094 is best suited for processing large numbers of rays.

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX SOURCE PROGRAM XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

```
$IBFTC MAIN
C      7094, FORTRAN IV, LINEAR-INTERPOLATION WAVE REFRACTION PROGRAM.
C      ORIGINAL PROGRAM BY GRISWOLD,NAGLE,AND MEHR. THIS PROGRAM ADAPTED
C      FROM ORIGINAL BY WILSON,HARRISON,KRUMBEIN,AND BENSON. 8/4/64.
       DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(100,82),C(4),XLABL(12)
       COMMON  S       , EM      , E       , YVW     , CMAT    , C
       COMMON  XLABL   , D       , TT      , CXY     , IT      , NGO
       COMMON  AMM     , ANN     , MAX     , NOT     , N       , MIT
       READ (5,5) ((S(I,J),J=1,3),I=1,3)
     5 FORMAT(6F12.8)
       READ (5,7) ((EM(L,I),L=1,4),I=1,3)
     7 FORMAT(12F6.2)
       READ (5,500) NOTT
   500 FORMAT (I3)
       DO 399 NOT=1,NOTT
       READ (5,400) XLABL
   400 FORMAT (12A6)
       READ (5,402) MM,NN,CHECK,TT,NOJ,D
   402 FORMAT (2I4,F3.0,7X,F5.1,I5,F4.1)
       AMM = MM-1
       ANN = NN-1
       IF(CHECK) 10,10,20
    10 READ (5,11) ((CMAT(I,J),I=1,MM),J=1,NN)
    11 FORMAT (5(F10.8,3X))
       GO TO 14
    20 J = 1
       READ (5,11) (CMAT(I,J),I=1,MM)
       DO 77 J=2,NN
       DO 77 I=1,MM
    77 CMAT(I,J) = CMAT(I,1)
    14 DO 15 N=1, NOJ
       READ (5,6) A,X,Y
     6 FORMAT (F7.2,2F6.2)
       MAX = 1
       WRITE (6,403) XLABL,TT,NOT,N,MAX,X,Y,A
   403 FORMAT (1H1,12A6/9H PERIOD =,F5.1,6H SEC.,,10H BATCH NO.,I3,9H, RA
      1Y NO.,I3,1H.//4X,3HMAX,6X,1HX,8X,1HY,8X,5HANGLE,4X,6HPCTDIF//
      2I7,2F9.2,F11.2)
       A=A*.0174532925
       CALL RAYN (X,Y,A)
    15 CONTINUE
   399 CONTINUE
       WRITE (6,9999)
  9999 FORMAT (18H1 THIS IS THE END.)
       CALL EXIT
       STOP
       END
```

```
C
$IBFTC RAYN
      SUBROUTINE RAYN (X,Y,A)
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(100,82),C(4),XLABL(12)
      COMMON  S      , EM      , E       , YVW     , CMAT    , C
      COMMON  XLABL  , D       , TT      , CXY     , IT      , NGO
      COMMON  AMM    , ANN     , MAX     , NOT     , N       , MIT
      CALL SURFCE (X,Y,A,FK)
    3 MAX=1+MAX
      IF (MAX - 800) 399,15,15
  399 CALL MOVE (X,Y,A,FK)
      IF (CXY) 15,15,396
  396 GO TO (397,395,15), MIT
  395 PUNCH 200, MAX
  200 FORMAT(33H CURVATURE APPROXIMATED FOR MAX =,I4)
  397 ANGLE=A*57.29577951
      IF(MOD(MAX,40)) 20,5,20
    5 WRITE (6,7) XLABL,TT,NOT,N
    7 FORMAT (1H1,12A6/9H PERIOD =,F5.1,6H SEC.,,10H BATCH NO.,I3,9H, RA
     1Y NO.,I3,1H.//4X,3HMAX,6X,1HX,8X,1HY,8X,5HANGLE,4X,6HPCTDIF//)
      PCTDIF = ABSF((C(3)-E(1)-E(2)-E(3))/C(3))*100.
      WRITE (6,12) MAX,X,Y,ANGLE,PCTDIF
   12 FORMAT (I7,2F9.2,F11.2,F10.1)
      GO TO (3,15),NGO
   15 WRITE (6,13)
   13 FORMAT (13H RAY STOPPED.)
      RETURN
      END
C
$IBFTC SURFCE
      SUBROUTINE SURFCE (X,Y,A,FK)
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(100,82),C(4),XLABL(12)
      COMMON  S      , EM      , E       , YVW     , CMAT    , C
      COMMON  XLABL  , D       , TT      , CXY     , IT      , NGO
      COMMON  AMM    , ANN     , MAX     , NOT     , N       , MIT
      I=X+1.
      J=Y+1.
      FI=I
      FJ=J
      XL=X+1.-FI
      YL=Y+1.-FJ
      IF (MAX-1) 1,1,4
    4 IF (ZI-FI) 1,2,1
    2 IF (ZJ-FJ) 1,3,1
    1 ZI = FI
      ZJ = FJ
      C(1)=CMAT(I,J)
      C(2)=CMAT(I+1,J)
      C(3)=CMAT(I+1,J+1)
      C(4)=CMAT(I,J+1)
```

```
      DO 318 II=1,3
      YVW(II) = 0.
      DO 318 L=1,4
  318 YVW(II) = YVW(II)+C(L)*EM(L,II)
      DO 319 II=1,3
      E(II) = 0.
      DO 319 JJ=1,3
  319 E(II) = E(II)+S(II,JJ)*YVW(JJ)
    3 CXY = E(1) + E(2)*XL + E(3)*YL
      FK = (E(2)*SIN(A)-E(3)*COS(A))/CXY
      RETURN
      END
C
$IBFTC MOVE
      SUBROUTINE MOVE (X,Y,A,FK)
      DIMENSION S(3,3),EM(4,3),E(3),YVW(3),CMAT(100,82),C(4),XLABL(12)
      COMMON   S        , EM       , E        , YVW      , CMAT     , C
      COMMON   XLABL    , D        , TT       , CXY      , IT       , NGO
      COMMON   AMM      , ANN      , MAX      , NOT      , N        , MIT
      IF (MAX - 2) 102,102,104
  102 FKBAR=FK
  104 MIT = 1
      DO 20 IT=1,20
   39 DELA=FKBAR*D
      AA=A+DELA
      ABAR=A+.5*DELA
      DELX = D*COS(ABAR)
      DELY = D*SIN(ABAR)
      XX=X+DELX
      YY=Y+DELY
      GO TO (101,6), MIT
  101 CALL SURFCE (XX,YY,AA,FKK)
      IF (CXY) 38,38,10
   10 FKBAR = 0.5 * (FK + FKK)
      IF (IT - 18) 5,37,9
   37 FKKPP = FKBAR
    5 IF (MAX - 2) 7,7,9
    7 IF (IT - 1) 20,20,9
    9 IF (ABS (FKKP-FKBAR) - (0.00009/D)) 6,6,20
   20 FKKP = FKBAR
      IF (ABS (FKKPP - FKBAR) - (0.00009/D)) 18,18,17
   17 MIT = 3
      GO TO 38
   18 FKBAR = 0.5 * (FKBAR + FKKP)
      MIT = 2
      GO TO 39
    6 NGO = 1
      IF ((XX-1.0)*((AMM-1.0)-XX))2,2,3
    3 IF ((YY-1.0)*((ANN-1.0)-YY))2,2,8
    2 NGO = 2
    8 X = XX
      Y = YY
      A = AA
      FK = FKK
   38 RETURN
      END
```

APPENDIX F

Derivations Relating $\frac{\partial C}{\partial X}$ with $\frac{\partial d}{\partial X}$, and $\frac{\partial C}{\partial Y}$ with $\frac{\partial d}{\partial Y}$

Given: $C = \frac{gT}{2\pi} \tanh \left[\frac{2\pi d}{CT}\right]$

Then: $\tanh\left[\frac{2\pi d}{CT}\right] = \frac{2\pi C}{gT}$

$$\frac{2\pi d}{CT} = \tanh^{-1}\left[\frac{2\pi C}{gT}\right]$$

$$d = \frac{CT}{2\pi} \cdot \frac{1}{2} \cdot \left[\ln\left(1 + \frac{2\pi C}{gT}\right) - \ln\left(1 - \frac{2\pi C}{gT}\right)\right]$$

Let: $k' = T/4\pi$ and $k'' = 2\pi/gT$

Then: $d = Ck' \left[\ln(1 + k''C) - \ln(1 - k''C)\right]$

Given: $C = f(d)$ and $d = g(X,Y)$

However, it may also be considered that:

$$d = F(C) \text{ and } C = G(X,Y)$$

Therefore (Kaplan, 1952, p.86):

$$\frac{\partial d}{\partial X} = \frac{d(d)}{dC} \cdot \frac{\partial C}{\partial X} \text{ and } \frac{\partial d}{\partial Y} = \frac{d(d)}{dC} \cdot \frac{\partial C}{\partial Y}$$

$$\frac{\partial d}{\partial X} = k'\left[C\frac{d}{dc}\left(\ln(1+k''C) - \ln(1-k''C)\right)\frac{\partial C}{\partial X} + \left(\ln(1+k''C) - \ln(1-k''C)\right)\frac{\partial C}{\partial X}\right]$$

$$\frac{\partial d}{\partial X} = k'\left[C\left(\frac{k''}{1+k''C} - \frac{(-k'')}{1-k''C}\right)\frac{\partial C}{\partial X} + \left(\ln(1+k''C) - \ln(1-k''C)\right)\frac{\partial C}{\partial X}\right]$$

$$\frac{\partial C}{\partial X} = \frac{\partial d}{\partial X} \cdot \frac{1}{k'}\left[\frac{1}{\frac{Ck''}{1+k''C} + \frac{Ck''}{1-k''C} + \ln(1+k''C) - \ln(1-k''C)}\right]$$

Similary:

$$\frac{\partial C}{\partial Y} = \frac{\partial d}{\partial Y} \cdot \frac{1}{k'}\left[\frac{1}{\frac{Ck''}{1+k''C} + \frac{Ck''}{1-k''C} + \ln(1+k''C) - \ln(1-k''C)}\right]$$

APPENDIX G

Method Used in Obtaining Most-Frequent Combinations of Deep-Water Height, Period, And Direction of Waves Capable of Striking Virginia Beach

A rough estimate of the 15 most-frequent combinations of wave height, period, and direction of deep-water waves capable of striking Virginia Beach is gained by an analysis of five representative years of wave observations at the Chesapeake Lightship (fig. 1). Results of the analysis appear in table G1 and indicate that only six combinations of period and direction are involved in the 15 combinations. These six combinations are rough approximations for input data, not only because of the crude methods involved in wave observation and recording, but also because it is assumed that one can preserve wave-direction angles for the waves observed at Chesapeake Lightship when searching for deep-water origin points that will yield rays capable of striking the target area. This is in error for some of the 4-second and all of the 6-second waves because they have already undergone refraction by the time they have reached Chesapeake Lightship; it is, therefore, invalid to select deep-water starting points for most waves by assuming that deep-water wave directions in water depths greater than 64 feet (Lightship water depth) are parallel to those observed at the Lightship.

The 4 and 6-second waves that dominate the Lightship observations of table G1 are a product of the convention used here in interpreting the Ship's International Code. The convention is based upon an analysis of wave gage records that indicate a dominant 4-second period (table G2) at a wave recording point off Cape Henry (fig. G1). (The Cape Henry wave gage was operated by the U. S. Navy between 1951 and 1956. It consisted

of a stepped-resistance wave staff; strip-chart recording were analyzed the significant wave method at 4-hour intervals.) Thus, code figure 2 Code Table 17 of the Ship's International Code, which stands for waves of 5-second period or less, was taken arbitrarily to represent waves of 4-second period, while code figure 3 (5 to 7 seconds) was taken as 6 seconds.

Table G1.-The fifteen most-frequently observed combinations of wave period, height, and direction (from 30°-150°T) as observed at Chesapeake Lightship during five representative years

Combination	Period* (sec)	Height (ft)	Direction from which wave front travels (°T)	No. of observations**
1	4	1	150	392
2	4	3	60	286
3	4	1	60	279
4	4	1	90	265
5	4	3	150	208
6	4	3	90	166
7	4	5	60	103
8	6	3	60	100
9	4	3	30	94
10	6	3	90	93
11	4	1	30	79
12	6	5	60	75
13	6	1	90	50
14	4	6	60	42
15	4	5	30	39

*Waves coded 2 (table 17, Ship's International Code) were considered as 4-sec waves; those coded 3 were considered 6-sec waves. See explanation in text and table G2.

**Total observations of waves capable of striking Virginia Beach (traveling from 30° through 150° True) = 3016

Table G2.-Portion of a frequency table for wave periods observed at the Cape Henry wave gage for a period of four representative years (each month completely represented 4 times)

Month:	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	TOT
No. Records (H 0.2 ft)	535	358	442	534	577	459	253	526	413	398	473	375	5363
T(sec)	Frequency (%)												Mean**
1.5			.4	.3	.1			.1					.1
2.0	.7	1.1	1.3	1.5	1.7	.8		1.1	.4	.5	.2		.8
2.5	4.3	2.5	3.6	3.7	3.2	2.4	.3	4.3	2.1	3.7	2.1	3.4	3.2
3.0	9.7	6.9	4.9	8.7	8.6	6.9	5.8	10.6	10.4	8.5	6.5	5.3	8.1
3.5	12.3	10.3	9.4	16.2	11.9	9.5	7.3	12.9	18.1*	18.5*	10.3	10.1	12.7
4.0	13.4*	12.0	12.4*	17.0*	16.2	14.6*	14.2*	17.4*	16.7	16.3	13.0*	12.2*	15.2*
4.5	11.7	13.1*	11.9	11.2	17.8*	14.7*	12.8	15.7	15.7	14.0	12.6	11.7	13.9
5.0	5.9	10.3	8.5	11.0	13.3	13.3	11.7	14.2	11.1	8.2	8.4	7.7	10.6
5.5	5.0	6.4	8.8	3.7	9.8	9.3	6.5	5.7	5.3	4.2	5.4	4.8	6.4
6.0	3.9	6.9	9.0	3.1	6.6	6.1	7.3	6.0	4.1	1.7	3.3	1.6	5.1
6.5	2.9	3.9	4.0	3.3	2.0	4.3	6.5	2.4	3.1	3.0	3.8	2.9	3.4
7.0	3.1	3.6	4.9	2.4	3.1	3.7	7.3	3.2	3.1	3.5	3.3	4.5	3.7
7.5	2.2	2.7	4.9	1.3	3.6	4.5	4.0	2.0	1.9	1.5	2.5	2.9	2.6
8.0	3.5	2.5	2.4	2.4	1.2	3.0	6.2	.5	1.4	2.5	4.2	5.0	2.8
8.5	4.6	4.4	2.9	3.7	.6	1.5	4.7	.7	1.2	1.5	5.4	6.9	3.1

*Modal class

**The mean of all periods for the complete set of observations is 5.3 sec.

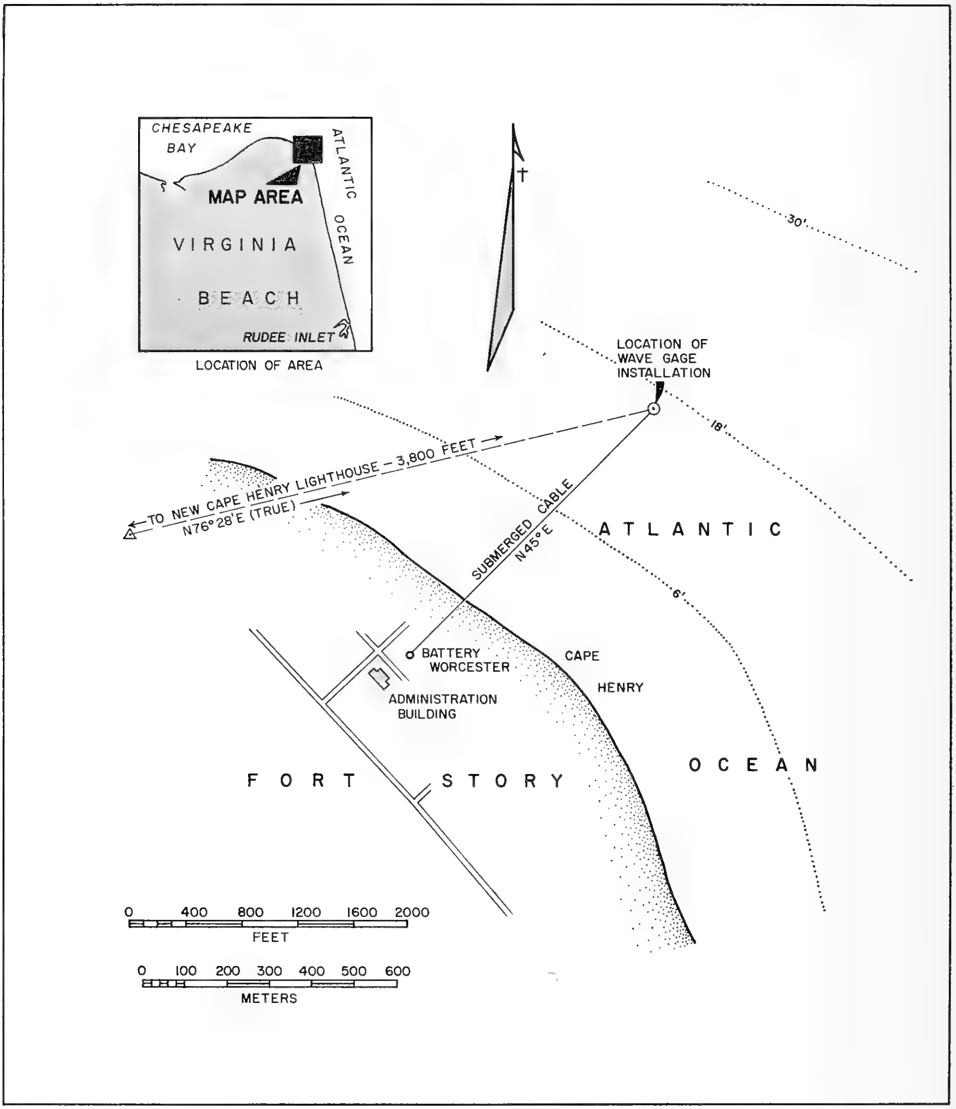


FIGURE G1 FORMER LOCATION OF U.S. NAVY WAVE GAGE OFF CAPE HENRY VIRGINIA, IN 20 FEET OF WATER

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4.

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II Wilson, W. S.
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